RESEARCH PLAN FOR THE PROJECT
"Generalizations of braid groups and knot theory in dimension 4"

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(1) **Towards a topological version of Markov's theorem for ribbon torus-links.** For this project the possible paths are two: the first one continuing in the spirit of the first part directly on the topology of closed loop braids, or considering Bennequin's proof of the classical case and use contact structures to prove a complete topological version of Markov's theorem for loop braids.

(2) **Extending representations of the braid group to the loop braid group.** The first step to take in the sense of this project will be considering the extension of Burau representation to LB_n defined on the welded braid group. Constructing the analogous of the 1-parameter Iwahori-Hecke algebra means considering the quotient of the algebra of the loop braid group by relations that give a finite-dimensional quotient in which loop braid relations and extended Burau matrices' relations are respected. Then I would need to find a linear basis for this quotient algebra in order to study Markov traces: this should for instance allow to recover the Alexander polynomial on ribbon torus-links. After that I will consider other quotients, as for example the analogous if the Temperley–Lieb algebras, finding the appropriate set of relations.

(3) **Braided ribbon surfaces in 4-space using quandles and biquandles, and their cohomology theory.** A quandle is a set with a self-distributive binary operation with the data of some axioms. In the 1990's and the early 2000's, the homology and cohomology theory for quandles appeared: one can derive diagrammatically a cohomology theory for quandles, coming from Reidemeister moves for classical knots and knotted surfaces. This is a tool to construct knotted objects invariants. In 2003 Carter, Jelsovsky, Kamada, Langford, and Saito used quandle cocycles to define statesum invariants for knots and links in dimension 3 and for knotted surfaces in dimension 4. It seems then natural to ask what are the application to the case of braided ribbon surfaces in the 4-dimensional space as are loop braids.

(4) **Extending knot contact homology to ribbon objects, and other applications of contact geometry.** This project is composed by a learning phase about contact topology, and two sub-projects. The strategy to extend knot contact homology to ribbon objects is to follow Ng work, study the possible obstructions to the adaptation to ribbon objects, and work around these obstructions. The same strategy will be applied to open book decompositions, by applying Pavelescu's techniques.

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