Research Plan

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A handlebody-link is the disjoint union of handlebodies embedded in the 3-sphere S^3. We want to establish new invariants of handlebody-links by means of partially multiplicative biquandles and also to understand handlebody-links through these newly established invariants. We are trying to construct biquandle cocycle invariants for handlebody-links.

A surface-link is a closed surface smoothly embedded in 4-space. A surface-link can be described by marked graph diagrams. By using marked graph diagrams, we get dual graph diagrams and want to construct surface biquasiles. By using this algebraic structure, we are trying to construct new invariants of surface-links.

An immersed surface-link is a closed surface smoothly immersed in 4-space. An immersed surface-link is a generalization of a surface-link. Similar to the surface-link case, immersed surface-links are represented by marked graph diagrams. If marked graph diagrams are related by a finite sequence of 8 types of local moves, then the surface-links represented by marked graph diagrams are equivalent. We want to find local moves corresponding to immersed surface-links.

The main body of this research proposal is as follows.

- **Extension biquandles of quandles**

We are going to find a canonical way to obtain a biquandle from a given quandle, which is an extension of the quandle in some sense.

- **Biquandle (co)homology and handlebody-links**

We defined partially multiplicative biquandles. By using this biquandle, we are going to construct partially multiplicative biquandle homology and cohomology. We are planning to introduce biquandle cocycle invariant of handlebody-links.

- **Surface Biquasiles**

D. Needell and S. Nelson introduced dual graph diagrams representing oriented knots and links. They gave an algebraic structure, called a biquasile, whose axioms are motivated by dual graph Reidemeister moves. We will introduce an algebraic structure, called a surface biquasile. By using this algebraic structure, we are going to introduce invariants of oriented surface-links.

- **Generalized Yoshikawa moves for immersed surface-links**

A marked graph diagram is a link diagram possibly with marked 4-valent vertices. S.J. Lomonaco, Jr. and K. Yoshikawa introduced a method of representing surface-links by marked graph diagrams. Two marked graph diagrams representing equivalent surface-links are related by a finite sequence of 8 types of local moves, called Yoshikawa moves.