Research Result

Jieon Kim

An immersed surface-link is a closed surface smoothly immersed in 4-space. When the smooth immersion is a smooth embedding, an immersed surface-link is called a surface-link. One of fundamental problems in the study of (immersed) surface-links is to find computable and effective invariants. The most common method of representing (immersed) surface-link is a broken surface diagram, which is a projection image of it in 3-space with over/under sheet information at each double point curve. Recently established method of representing a (immersed) surface-link is a marked graph diagram in the plane, which is a classical link diagram possibly with marked 4-valent vertices. Using marked graph representation of (immersed) surface-links, we have the following results.

- **Generating sets of Yoshikawa moves for marked graph diagrams**

K. Yoshikawa introduced local moves on marked graph diagrams. A collection S of Yoshikawa moves is called a generating set of Yoshikawa moves if any Yoshikawa move is obtained by a finite sequence of the moves in the set S. We give some generating sets of Yoshikawa moves on marked graph diagrams representing surface-links and show independence of certain Yoshikawa moves from the other moves.

- **Ideal coset invariants for surface-links**

Using a state-model involving a given classical link invariant, S.Y. Lee introduced a polynomial [[D]] for a marked graph diagram D. We introduce an ideal coset invariant for surface-links, which is defined to be the coset of the polynomial [[D]] in a quotient ring of a certain polynomial ring modulo some ideal and represented by a normal form (a unique representative for the coset of [[D]]) that can be calculated from [[D]] with help of a Gröbner basis package on computer.

- **Computations of quandle cocycle invariants of surface-links using marked graph diagrams**

Using the cohomology theory of quandles, quandle cocycle invariants of oriented surface-links are defined by J.S. Carter, D. Jelsovsky, S. Kamada, L. Langford and M. Saito via broken surface diagrams. J.S. Carter, S. Kamada, and M. Saito defined shadow quandle cocycle invariants of oriented surface-links, generalizations of quandle cocycle invariants. S. Kamada and K. Oshiro introduced the symmetric quandle cocycle invariant for unoriented surface-links via broken surface diagrams. We provide methods of computing them from a given marked graph diagram.

- **Alexander biquandles of oriented surface-links via marked graph diagrams**

T. Carrell defined the fundamental biquandle of oriented surface-links by a presentation obtained from its broken surface diagram, which is an invariant up to isomorphism of the fundamental biquandle. S. Ashihara gave a method of calculating the fundamental biquandles from marked graph diagrams. We deal with computable invariants obtained from the fundamental Alexander biquandles. By using these invariants, non-invertibility of orientable surface-links are discussed.

• **A multiple conjugation biquandle and handlebody-links**

A. Ishii defined a multiple conjugation quandle. A handlebody-link is the disjoint union of handlebodies embedded in the 3-sphere S^3. We introduce a multiple conjugation biquandle, which is a generalization of a multiple conjugation quandle, and show that it is the universal algebra to define a semi-arc coloring invariant for handlebody-links. We extend the notion of n-parallel biquandle operations for any integer n, and show that any biquandle gives a multiple conjugation biquandle with them.

• **Immersed 2-knots with essential singularity**

An immersed surface-link with essential singularity is an immersed surface-link which is not equivalent to the connected sum of any surface-link and any unknotted immersed sphere, where an unknotted immersed sphere is an immersed surface-link represented by D or D'. We show that there are infinitely many immersed 2-knots with essential singularity, that is, infinitely many immersed 2-knots which are not equivalent to the connected sum of any 2-knot and any unknotted immersed sphere.

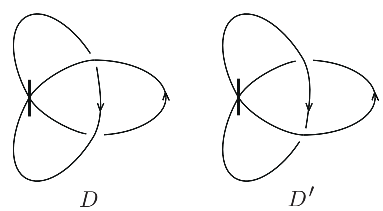

Figure 1: Unknotted immersed spheres

• **Biquandle cohomology and state-sum invariants of knots and surface-links**

J.S. Carter, M. Elhamdadi and M. Saito introduced the cohomology theory of biquandles and biquandle cocycle invariants of oriented links by using the cohomology theory of biquandles. We define the biquandle cocycle invariants of oriented surface-links via broken surface diagrams or marked graph diagrams of oriented surface-links.