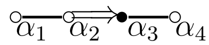
Research Plan

Now we consider following problems:

1. Research on non-naturally reductive Einstein metrics on exceptional Lie group F_4

We consider the flag manifold G/K with Painted Dynkin diagram as following



Let $\mathfrak{g}$ and $\mathfrak{k}$ be the Lie algebras of G and K respectively. Then we have the following left-invariant metric on G which is $\mathrm{Ad}(K)$-invariant:

$$\langle\cdot,\cdot\rangle = u_0 \cdot B \mid_{\mathfrak{k}_0} + u_1 B \mid_{\mathfrak{k}_1} + u_2 B \mid_{\mathfrak{k}_2} + x_1 \cdot B \mid_{\mathfrak{m}_1} + x_2 \cdot B \mid_{\mathfrak{m}_2} + x_2 \cdot B \mid_{\mathfrak{m}_3}, \tag{1}$$

where $\mathfrak{k} = \mathfrak{k}_0 \oplus \mathfrak{k}_1 \oplus \mathfrak{h}_2$ be the decomposition into ideals of $\mathfrak{k}$, $\mathfrak{k}_0$ is the center of $\mathfrak{k}$ and $\mathfrak{k}_i (i = 1, 2)$ are simple ideas of $\mathfrak{k}$, $A_0 \mid_{\mathfrak{k}_0}$ be an arbitrary metric on $\mathfrak{k}_0$, and $\mathfrak{m} = \mathfrak{m}_1 + \mathfrak{m}_2 + \mathfrak{m}_3$ be the isotropy irreducible decomposition of $\mathfrak{m} = T_o(G/K)$.

By the Theorem A in [2] we get the non-zero structure constants of F_4, then by computing Gröbner basis of the system of Einstein equations of F_4 we obtain all the Einstein metrics on F_4. We give the conditions that non-naturally reductive Einstein metrics need to satisfy, then we could obtain non-naturally reductive Einstein metrics on Lie group F_4.

2. Research on two-step g.o. spaces

We plan to investigate two-step g.o. spaces, especially for the case of left-invariant metrics. Firstly we could consider what happens for some particular Lie groups, for examples, three-dimensional solvable Lie groups.

First we should consider how to define metrics on these spaces, then we plan to give necessary condition, or sufficient condition, or necessary and sufficient condition that the g.o. metrics satisfy. Finally we consider the conditions that two-step g.o. spaces need to be met.

3. Research on homotopy groups of flag manifolds

We plan to compute homotopy groups of flag manifolds G/K. Firstly we consider homotopy groups of flag manifolds $SU(n)/(U(1)\times\cdots\times U(1)\times SU(n_1)\times\cdots\times SU(n_s))$, write $S = U(1) \times \cdots \times U(1)$, $K_1 = SU(n_1) \times \cdots \times SU(n_s)$ and $K = S \times K_1$. Using fiber spectrum sequences $G/K \to G/(S \times K_1) \to S$, we get $\Pi_k(G/K) = \Pi_k(G/K_1)$. This implies that we reduce to compute homotopy groups of M-spaces G/K_1. Then we give several fiber spectrum sequences and corresponding exact sequences of homotopy group, we could get the homotopy groups of flag manifolds $SU(n)/(U(1)\times\cdots\times U(1)\times SU(n_1)\times\cdots\times SU(n_s))$. By the same method we compute homotopy groups of flag manifolds G/K for $G = SO(n), Sp(n), E_6, E_7, E_8, F_4, G_2$.