Future research plan

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The research topics can be broadly summarized into the following three parts:

(1) Bicategorical covering theory for derived equivalences,
(2) Derived equivalence classifications, and
(3) Decomposition theory of modules (CREST research project).

(1) aims to generalize the covering theory under actions of a group that has been usded in the derived equivalence classification to the covering theory under lax actions of a small category; (2) aims to use covering theory for derived equivalences to give derived equivalence classifications for some classes of algebras that are controlled by coverings; the aim of (3) is to give the basic theory and algorithms needed for the indecomposable decomposition of persistence modules, which is needed in CREST research "Construction of theory of topological data analysis aiming to creation of discription language for soft matter'. Due to the limitation of space, only (1) and (2) are described below, and (3) is omitted.

(1) Let G be a group. The covering theory in representation thoery of algebras was formlated by Gabriel as an algebraic form of a covering functor between linear categories that are presented by quivers with relations. In the paper [20] we generalized this algebraic formulation to a G-covering functor between arbitrary linear categories by using a family of natural transformations, which gave a new tool to induce derived equivalences. Namely, for two G-covering functors $\mathcal{C} \to \mathcal{B}$ and $\mathcal{C}' \to \mathcal{B}'$, we gave (a) a sufficient condition for a derived equivalence between $\mathcal{C}$ and $\mathcal{C}'$ to induce one between $\mathcal{B} \simeq \mathcal{C}/G$ and $\mathcal{B}' \simeq \mathcal{C}'/G$. Conversely we could recently give (b) a sufficient condition for a derived equivalence between $\mathcal{B}$ and $\mathcal{B}'$ to induce one between $\mathcal{C} \simeq \mathcal{B}\#G$ and $\mathcal{C}' \simeq \mathcal{B}'\#G$. Here let $\Bbbk$ be a commutative ring, and set $\Bbbk$-**Cat** to be the 2-category consisting of the small $\Bbbk$-categories, the functors between them and the natural transformations between those functors. In the paper [22] we genearalized (a) to the case of 2-categorical coverig theory under colax actions of a small category I (that is, colac functors $X\colon I \to \Bbbk\text{-}\mathbf{Cat}$), where the orbit category construction is generalaized to the Grothendieck construction $\mathrm{Gr}(X)$. This can be seen as a $\Bbbk$-category obtained by glueing the $\Bbbk$-cagegories $X(i)$ $(i \in I)$ together. On the other hand, the concept of smash products is extended to the case of colax actions of I by Dai Tamaki. We are planning to generalize the result of (b) to the case of colax actions of I, which is now almost complete. Further, let $\Bbbk\text{-}\mathbf{Cat}^{\mathrm{b}}$ be the bicategory consisting of the small $\Bbbk$-categories, the bimodules over them, and the morphisms between those bimodules. Then it is also possible to define the Grothendieck construction for lax functors $X\colon I \to \Bbbk\text{-}\mathbf{Cat}^{\mathrm{b}}$, which makes it possible to construct much wider range of $\Bbbk$-categories. The result of (a) is being extended to this setting, and we will complete them. We also plan to generalize the result of (b) to this setting as well.

(2) Let A be an algebra of finite global dimension, $\hat{A}$ the repetitive category of A, and g an automorphism of $\hat{A}$ such that the number of g-orbits of objects in $\hat{A}$ is finite. Then the orbit category $\hat{A}/\langle g\rangle$ turns out to be a selfinjective algebra. These form a very important class of selfinjective algebras, which includes important blocks of group algebras. The main aim of this subject to classify this class under derived equivalences. There are two different theoretical treatments depending on the form of g: (i) g is the product of a power of the Nakayama automorphism of $\hat{A}$ and an automorphism r of $\hat{A}$ satisfying $r(A^{[0]}) \subseteq A^{[0]}$, where $A^{[0]}$ is the 0-th part of $\hat{A}$; (ii) a power of g is is the Nakayama automorphism of $\hat{A}$. Blocks of group algebras are of type (ii). So far, we have completed the classification for the class of algebras of type (i) where A is derived equivalent to a hereditary algebra of tree type (see [15, 24]). We are now trying to complete the classification for the classe of algebras of type (i) when A is derived equivalent to a hereditary algebra of type $\tilde{A}_n$. In this case, a theorem has already been proposed by Bocian–Skowroński, but we found an error in their classification. This, together with the results of the paper [24], completes the derived equivalence classification for the class of algebras where A is derived equivalent to a hereditary algebra of Euclidean type. Besides this, we will also classify the class of algebras of type (ii) using the covering theory developed in the study (1). Once this is complete, we will be able to deal with derived equivalences, especially between blocks of group algebas, which will have applications to Broué's abelian defect conjecture.