Previous research

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The beautiful theory of the elliptic functions is applied to many fields in mathematics and science. As the technology advances, there are movements to apply the Abelian functions associated with more general curves, which are generalizations of the elliptic functions to many variables, to many fields in mathematics and science. For example, in cryptography, the theory of Abelian functions is applied to the hyperelliptic curve cryptography, which is a generalization of the elliptic curve cryptography. In theory of functions, the theory of Abelian functions is applied to the theory of the conformal maps from the complex upper half plane to polygons, which are called Schwarz-Christoffel mappings. Therefore, it is very important to construct the concrete theory of Abelian functions like the theory of the elliptic functions.

The sigma functions play important roles in the theory of Abelian functions. The elliptic sigma functions, which are defined and studied by Weierstrass, are generalized to the multivariable sigma functions associated with the hyperelliptic curves by F. Klein. About 100 years ago, H. Baker developed the theory of the hyperelliptic sigma functions. Recently, V. Buchstaber, V. Enolski, and D. Leykin generalized the hyperelliptic sigma functions to a plane curve called plane telescopic curve. In this research, we extended the sigma functions to the general telescopic curves. Further, we generalized the famous Jacobi inversion formulae to the telescopic curves.

The field of rational functions on elliptic curves is an algebraic object and one can determine the structure of this field explicitly. By using the Abel-Jacobi map, one can determine the structure of the field of meromorphic functions on the Jacobian of elliptic curves, which is an analytic object. Baker extended this method and determined the structure of the field of meromorphic functions on the Jacobian of hyperelliptic curves explicitly. Thus, Baker determined the defining equations of the Jacobian of hyperelliptic curves. The zero set of the sigma functions in the Jacobian is called a sigma divisor. In this research, for the hyperelliptic curve of genus 3, we determined the structure of the field of meromorphic functions on the sigma divisor. Thus we determined the defining equations of the sigma divisor for the hyperelliptic curve of genus 3. As an application of this result, we derived partial differential equations, which can be called two parametric deformed KdV-equations, integrable by the meromorphic funcitons on the sigma divisor of the hyperelliptic curve of genus 3.

In this research, we derived the recurrence formulae with respect to the coefficients of the series expansion of the inverse function of the hyperelliptic integrals of genus 2. When the curves of genus 2 deform to elliptic curves, we showed that the inverse function transforms into the Weierstrass elliptic function. By this result, a new relation on the degeneration of the Abelian functions under the degeneration of the curves was given.

Let $f(x)$ be a polynomial of degree 5, V be the hyperellptic curve of genus 2 defined by $y^2 = f(x)$, and $\sigma(u)$ be the sigma funciton associated with V. Via the Abel-Jacobi map, the rational function y on V can be identified with meromorphic functions F and G on $\mathbb{C}^2$, which are defined by the sigma function. Then there exists a meromorphic function H on $\mathbb{C}^2$ such that the relation $F - G = \sigma H$ holds on $\mathbb{C}^2$. In this research, we gave an explicit expression of H in terms of σ. In theory of functions, it is an important problem to decompose a multivariable meromorphic function into a product of two meromorphic functions. In this research, we gave a non-trivial example for this problem.