

## Research results

Let  $p$  be an odd prime. Triple product  $L$ -functions attached to triplets of modular forms have Euler products and functional equations. We have studied  $p$ -adic triple product  $L$ -functions which are  $p$ -adic analytic functions interpolating an infinite number of central critical values of triple product  $L$ -functions attached to triplet of modular forms.

### Proceeding research

Let  $f$  be an ordinary cusp form. Haruzo Hida constructed a Hida family  $F$  attached to  $f$  which was a  $p$ -adic family of ordinary cusp forms. The Hida family  $F$  which interpolates  $f$  is unique. Let  $(f, g, h)$  be a triple of ordinary cusp forms. Then there exists the triple  $(F, G, H)$  of Hida families attached to  $(f, g, h)$ . Ming-Lun Hsieh constructed a  $p$ -adic triple product  $L$ -function attached to  $(F, G, H)$  in [Hsi17]. As a generalization of Hida family, Coleman families were defined in [Corollary B5.7.1, Col97]. Let  $f$  be a cusp form which is not ordinary. Then we can construct a Coleman family  $F$  which interpolates  $f$  uniquely.

### Main results

Let  $(f, g, h)$  be a triple of cusp form of weight  $(k, l, m)$ . If there exists a triangle with sides  $(k, l, m)$ , we say that the triple  $(f, g, h)$  satisfies a balanced condition. Otherwise, we say that the triple  $(f, g, h)$  satisfies an unbalanced condition. If a  $p$ -adic triple product  $L$ -function interpolates central critical values of triple product  $L$ -functions attached to triples of cusp forms satisfying the balanced condition (resp. unbalanced condition), the  $p$ -adic triple product  $L$ -function is called the balanced (resp. unbalanced)  $p$ -adic triple product  $L$ -function. In [Fuk19], we generalized his results [Hsi17] under unbalanced  $p$ -adic triple product  $L$ -functions. Let  $F$  be a Hida family and  $G, H$  general  $p$ -adic families of cusp forms. We constructed a unbalanced  $p$ -adic triple product  $L$ -function attached to  $(F, G, H)$ . In the previous research [Hsi17], he constructed unbalanced  $p$ -adic triple product  $L$ -functions attached to triples of Hida families. In our result, we can take more general  $p$ -adic families for  $G, H$ . For example, we can take Hida families, Coleman families or CM-families as an examples of  $G, H$ .

### Reference

- [Col97] R. F. Coleman,  $p$ -adic Banach spaces and families of modular forms. *Invent. Math.*, 127(3):417-479, 1997.
- [Fuk19] K. Fukunaga, Triple product  $p$ -adic  $L$ -function attached to  $p$ -adic families of modular forms, arxiv:1909.03165.
- [Hsi17] M.-L. Hsieh, Hida families and  $p$ -adic triple product  $L$ -functions, *AJM*, to appear.