SUMMARY of RESEARCH

My main field of interest is number theory. The last several years I worked on explicit formulas of Shintani functions and their applications to L-functions associated with automorphic forms.

(1) Complex Shintani Functions for $(GL_2, GL_1 \times GL_1)$

Hirano proved an explicit formula of Shintani functions on $GL_2(\mathbb{C})$ associated with unramified principal series representations with trivial $U(2)$-type. In the general situation, however, explicit formulas of Shintani functions are unknown. I proved an inductive formula to construct Shintani functions with *non-trivial* $U(2)$-type from those with *trivial* $U(2)$-type by using Zuckerman tensoring. As a by-product, I found that some Heun's differential equations have solutions which are expressed as linear combinations of Gauss hypergeometric functions with polynomial coefficients. It is said that, in general, analysis of solutions of Heun's differential equations is difficult. Because they have accessory parameters. So the result is interesting from the point of view of the theory of Special functions.

(2) Unramified Shintani Functions for $(GSp_4, GSpin_4)$

I established an explicit formula of Shintani functions for GSp_4 over the non-archimedean local field of general characteristic by following in the footsteps for Shintani functions on SO_n of Murase–Sugano. The important point in this research is that the proof of meromorphic continuation of Shintani functionals, which is key lemma, was simplified by using Bernstein's rationality theorem, which is easily proved. This method is also applicable to Shintani functions on general reductive groups. I formulated the local integral of Murase–Sugano type for GSp_4, and as an application of the explicit formula proved that the local integral represents the spin L-factor of GSp_4.

(3) Global Murase–Sugano Integrals for $(GSp_4, GSpin_4)$

I formulated a global zeta integral of Murase–Sugano type for the triple $(GSpin_6, GSpin_5 = GSp_4, GSpin_4)$ of reductive groups, and proved the basic identity.

(4) Real Shintani Functions for $(GSp_4, GSpin_4)$

I proved that the real Shintani function associated with the holomorphic discrete series representation of $GSp_4(\mathbb{R})$ is expressed by using generalized hypergeometric functions ${}_3F_2$. Moreover I computed the local zeta integral of Murase–Sugano type by using the explicit formula. As a result, I found that the local zeta integral represents the spin L-factor associated with the holomorphic discrete series representation of $GSp_4(\mathbb{R})$.

(5) Special Values of Tensor Product L-functions and Triple Product L-functions

I proved an explicit formula for the critical values of the Rankin–Selberg L-function associated with two modular forms by giving an explicit formula for all the Fourier coefficients of the holomorphic projection of nearly holomorphic modular forms. More precisely, I proved that the critical value of the Rankin–Selberg L-function is expressed as a finite sum using Bernoulli numbers and Fourier coefficients of cusp forms in the orthogonal basis. In addition, as a corollary of its proof, I proved that a cusp form g is uniquely determined by certain critical values of the family of Rankin–Selberg L-series $D(s, f \otimes g)$, where f runs over a fixed orthogonal basis of cusp forms. Also, applying the above method to the triple product L-function, I and Kengo Fukunaga proved explicit formulas for the rightmost critical value and the central critical value of the triple product L-function. I was in charge of overall supervision, computing the rightmost critical value and recording research progress.