Research Plan

Let $\mathfrak{g}$ be an affine Lie algebra. For $\mathfrak{g}$ of nonexceptional types, the Kerov-Kirillov-Reshetikhin (KKR) type bijection Φ from rigged configurations to the highest weight paths, which are tensor products of Kirillov-Reshtikhin (KR) crystals associated with $\mathfrak{g}$, is established and a uniform description of the bijection Φ is also given [1, 3]. On the other hand, there are several unsolved problems on the KKR type bijection Φ for $\mathfrak{g}$ of exceptional types. In particular, the explicit form of KKR type bijection is not known for the adjoint crystal of $\mathfrak{g} = G_2^{(1)}$, which is one of the affine Lie algebras of untwisted non-simply-laced types. T. Scrimshaw posed the following problem [3].

Problem. Describe explicitly the map δ_θ for $\mathfrak{g}$ of untwisted non-simply-laced affine type.

Here, δ_θ is a building block of the bijection Φ. This problem was posed four years ago. Since then it had been a longstanding open problem. In 2019 the author gave heuristically a conjecture on the explicit form of Φ for the adjoint KR crystal of $\mathfrak{g} = G_2^{(1)}$. In 2020 he performed the large-scaled computer experiments and gave an affirmative proof on this conjecture. The research plan in 2021 consists of the following two.

(1) Publishing the paper on the solution of this problem.

(2) Attempting the further developments on the established KKR type bijection for the adjoint crystal of $\mathfrak{g} = G_2^{(1)}$.

The content of (2) is as follows. It is established that the rigged configuration admits a crystal structure [2]. Since the KKR type bijection for the adjoint crystal of $\mathfrak{g} = G_2^{(1)}$ has been established, it is now possible to clarify the corresponding crystal structure on rigged configurations, which is the goal of (2).

References

[1] M. Okado, A. Schilling, and T. Scrimshaw, Rigged configuration bijecttion and proof of the $X = M$ conjecture for nonexceptional affine types, J. Algebra **516** (2018) 1–37.

[2] A. Schilling, Crystal structure on rigged configurations, Int. Math. Res. Not. **2006** (2006) Art. ID 97376 1–27.

[3] T. Scrimshaw, Uniform description of the rigged configuration bijection, arXiv:math/1703.08945v1.