# Future research plans (Masataka Iwai)

By Research A and B, we know the structure of $X$ if the tangent bundle $T_X$ is positive. By the studies of Cao et al, we also know the structure of $X$ if the anticanonical divisor $-K_X$ is positive. From the above, we already know the structure of smooth algebraic varieties if $T_X$ or $-K_X$ is positive.

On the other hand, in the minimal model theory, which is a classification theory of algebraic varieties, it is not enough to consider the categories of smooth algebraic varieties, and it is necessary to consider singular varieties. However, we can not apply complex geometric methods to singular varieties, since singular varieties are not always complex manifolds.

To solve this problems, Campana proposed "research methods for singular varieties by using orbifolds". It is a method to investigate singular varieties by putting the orbifold structures in singular varieties and by applying usual complex geometric methods to orbifolds. In future researches, based on the proposal of Campana, I will investigate "geometric structures of singular varieties from the viewpoint of orbifolds". The details are described below.

**Research C. The structure theorem of singular varieties with positive tangent bundles.**

From Research A, if a tangent bundle $T_X$ is pseudo-effective, then $X$ is composed of rationally connected varieties and Abelian varieties. On the other hand, it is difficult to define rationally connectedness for singular varieties. Therefore we can not apply the usual methods to singular varieties. In Research C, I will define rationally connectedness for singular varieties and will show the structure theorem of singular varieties with positive tangent bundles.

Campana proposed "slope rationally connectedness", which is a version of rationally connectedness of orbifolds. From this point of view and Research A, it is expected that the singular varieties with pseudo-effective tangent bundle are composed of slope rationally connected varieties and the Abelian varieties. We will study research C by applying the theory of slope rational connectedness proposed by Campana, and by extending the foliation theory used in Study B to orbifold structures.

**Research D. Research on the fundamental groups of singular varieties.**

Wang established the structure theorem for singular varieties with nef anticanonical divisors, given the assumptions in the fundamental group. The assumption is that the fundamental group $\pi_1(X_{reg})$ of a regular locus $X_{reg}$ is polynomial growth if the singular variety $X$ has a nef anticanonical divisor. Wang conjectured that this assumption always holds. In Research D, I will work on this conjecture and will complete the structure theorem of singular manifolds with nef anticanonical divisors.

First, we will put the orbifold structure $\mathcal{X}$ on a singular variety $X$. we will solve the Monge-Ampere equation on the orbifold $\mathcal{X}$ and construct a Kähler metric with almost positive Ricci curvature. Then, by applying Bishop-Gromov's theorem on orbifold, we will show that the orbifold fundamental group $\pi_1^{orb}(\mathcal{X})$ is polynomial growth.