Summaries of researches and results (Masataka Iwai)

I outline my papers [3][4][5] on my list of publications. My major is **complex algebraic geometry.** I research about a classification of algebraic varieties (closed sub-manifolds in $\mathbb{CP}^N$) by using methods of **algebraic geometry, complex geometry and several complex analysis.**

Research A. The structure theorems of varieties with positive tangent bundles.

It is known that the structures of algebraic varieties are restricted if tangent bundles T_X or anticanonical divisors $-K_X := \det T_X$ are positive. A typical example is Mori's theorem: "If T_X is ample, then X is biholomorphic to $\mathbb{CP}^n$".

$$\begin{tikzcd} \boxed{\text{ample}} \arrow[r, Rightarrow] \arrow[d, Rightarrow] & \boxed{\text{nef}} \arrow[d, Rightarrow] \\ \boxed{\text{big}} \arrow[r, Rightarrow] & \boxed{\text{pseudo-effective}} \end{tikzcd}$$

In addition to ample, the concept of positivity used in algebraic geometry includes nef, big, and pseudo-effective, and there is a relationship as shown in the table on the right. Therefore, even if T_X is nef, big, or pseudo-effective, the structure of algebraic varieties is expected to be restricted. In fact, Campana et al. proved that if T_X is nef, then X consists of Fano varieties (varieties with ample aniticanonical divisors) and Abelian varieties (tori). In Research A, I studied the structures of algebraic varieties when T_X is big or pseudo-effective. We summarize including previous researches, and it is as follows. (The paper [3] is a joint work with Shin-ichi Matsumura and Genki Hosono in Tohoku University.)

$$\begin{tikzcd} \boxed{\text{[Mori]}T_X \text{ is ample} \Rightarrow X \cong \mathbb{CP}^n} \arrow[r] \arrow[d] & \boxed{\text{[Campana]}T_X \text{ is nef} \Rightarrow \text{Fano+Abelian}} \arrow[d] \\ \boxed{\text{[4]}T_X \text{ is big} \Rightarrow X \cong \mathbb{CP}^n} \arrow[r] & \boxed{\text{[3]}T_X \text{ is pseudo-effective} \Rightarrow \text{Rationally Connected+Abelian}} \end{tikzcd}$$

It is well known that $\mathbb{CP}^n$ is Fano and that Fano varieties are rationally connected. Therefore, Research A is a genelarizarion of previous studies. The classification of algebraic varieties with positive tangent bundles has been completed, since pseudo-effective is the weakest positivity used in algebraic geometry.

Research B. The structure theorems of varieties whose tangent bundles contain positive subbundles.

Peternell proposed that the structure of X will be restricted even if the subbundle $\mathcal{F}$ of T_X has positivity (ample, nef, and so on). In fact, Andreatta-Wisnieski proved that if $\mathcal{F}$ is ample, then X is biholomorphic to $\mathbb{CP}^n$. In Research B, I studied the structure of X if T_X contains a positive subbundle foliation $\mathcal{F}$. We summarize including previous researches, and it is as follows.

$$\begin{tikzcd} \boxed{\begin{array}{c}\text{[Andreatta-Wisnieski] } \mathcal{F} \text{ is ample} \\ \Rightarrow X \cong \mathbb{CP}^n\end{array}} \arrow[r] \arrow[d] & \boxed{\begin{array}{c}\text{[5] } \mathcal{F} \text{ is nef} \\ \Rightarrow \text{Fano+"Varieties with flat foliations"}\end{array}} \arrow[d] \\ \boxed{\begin{array}{c}\text{[5] } \mathcal{F} \text{ is big} \\ \Rightarrow \mathbb{CP}^r\text{+"Varieties with flat foliations"}\end{array}} \arrow[r] & \boxed{\begin{array}{c}\text{[5] } \mathcal{F} \text{ id pseudo-effective} \\ \Rightarrow \text{Rationally Connected+"Varieties with flat foliations"}\end{array}} \end{tikzcd}$$

"Varieties with flat foliations" are classified by Druel et al. In particular, if T_X is flat, then X is a finite quotient of Abelian varieties. Therefore Research B is a generalization of the structure theorem in Research A.