My achievements

◆ Background

· calibration : The calibration was introduced by Harvey and Lawson, which is a special closed k-form on a Riemannian manifold (M, g). It defines k-dimensional minimal submanifolds called **calibrated submanifolds**. The theory is closely related to that of holonomy groups. If (M, g) has special holonomy $\mathrm{Hol}(g)$, we can define canonical calibrations. Some examples are shown below. I am interested in the cases of G_2 and $\mathrm{Spin}(7)$.

$\mathrm{Hol}(g)$ $(\subset)$	$\mathrm{U}(n)$	$\mathrm{SU}(n)$	G_2	$\mathrm{Spin}(7)$
(M, g)	Kähler mfd.	Calabi-Yau mfd.	G_2-mfd.	Spin(7)-mfd.
calibrated submanifolds	complex submanifolds	special Lagrangian submanifolds	(co)associative submanifolds	Cayley submanifolds

· mirror symmetry : The "mirror" of a calibrated submanifold is a Hermitian connection (satisfying certain PDEs) of a complex Hermitian line bundle on a manifold with special holonomy. The mirror of a special Lagrangian, (co)associative, Cayley submanifold is called a **deformed Hermitian–Yang–Mills (dHYM)** , **G_2-deformed Donaldson–Thomas (G_2-dDT)** , Spin(7)**-deformed Donaldson–Thomas (**Spin(7)**-dDT)** connection, respectively. As the names indicate, they can also be considered as analogies of Hermitian Yang–Mills connections and Donaldson–Thomas connections (G_2, Spin(7)-instantons), respectively.

◆ My achievements

(I) Research of calibrated submanifolds :

(Ia) Construction of explicit examples : Explicit examples are very important because they provide local models of singularities and are useful for the desingularization by the gluing construction. However, calibrated submanifolds are defined by nonlinear PDEs, which makes the construction very difficult. In my paper [1], I constructed examples of special Lagrangian submanifolds in a toric Calabi-Yau manifold by applying the moment map technique on $\mathbb{C}^n$ by Joyce. In my papers [2,9,14], I explicitly constructed coassociative submanifolds in an asymptotically conical G_2-manifold by the symmetry of Lie groups. I classified homogeneous coassociative submanifolds and gave many cohomogeneity one examples. In particular, I could find examples with conical singularities and their desingularizations.

(Ib) Explicit deformations of calibrated cone submanifolds : To understand the conical singularities, we need to study deformations of calibrated cone submanifolds. Since there are no deformation theories of noncompact submanifolds in general, I considered deformations of explicit cones. In my papers [4,6,13], I studied the infinitesimal deformations (the "first order approximations" of deformations) of explicit homogeneous Cayley cones by representation theory. Then by geometric considerations, I clarified the local structures of moduli spaces of some homogeneous Cayley cones. Moreover, in my paper [7], I studied the "second order approximations" of deformations and clarified the local structures of moduli spaces more.

(II) Topology and moduli spaces of G_2, Spin(7)-manifolds :

In my papers [8,10], I defined geometric cohomologies of G_2, Spin(7)-manifolds using an algebraic structure called the Fr ölicher Nijenhuis bracket and determined their structures. In my paper [11], I developed these further and found new topological obstructions for a manifold to be a G_2- or Spin(7)-manifold from a more general consideration of the formality of differential graded algebras. In my paper [12], I found the common properties of moduli spaces of various geometric structures. As an application, I showed that two canonical Riemannian metrics on the G_2 moduli space have different metric completions. I also investigated the difference in the structure of the completions of the space of Riemannian metrics with respect to the metrics conformal to the Ebin metric.

(III) Moduli theory of dHYM and G_2-dDT connections :

In my paper [16], I studied the deformations of dHYM and G_2-dDT connections. Introducing a new balanced Hermitian structure and a new coclosed G_2-structure, I showed that each deformation is controlled by a subcomplex of an elliptic complex. Moreover, in the case of dHYM, I showed that the moduli space is always a finite dimensional orientable smooth manifold. In the case of G_2-dDT, I showed that this is the case if we perturb the G_2-structure. In my paper [18], we show similar statements for Spin(7)-dDT connections. Since the moduli spaces of special Lagrangian, associative and Cayley submanifolds have similar properties, it turns out that the local properties of moduli spaces are indeed preserved by mirror correspondence.