# Results

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### (1) Diagrams of surface-links and Roseman moves

A *surface-link* is a closed surface embedded in $\mathbb{R}^4$. A *diagram* of a surface-knot is its image via a generic projection from $\mathbb{R}^4$ to $\mathbb{R}^3$, equipped with over/under information. It is known that two surface-links are equivalent if and only if their diagrams are related by a finite sequence of seven types of local moves, called *Roseman moves*. By investigating geometrical information of surface-link diagrams, I solve an independence problem of seven types of Roseman moves as a local move. Moreover, jointly working with Professors Tanaka and Oshiro, we can obtain a result about an independence problem of the Roseman move called the *tetrahedral move* as a global move.

### (2) A generating set of oriented Roseman moves

Roseman moves are seven types of local modifications for surface-link diagrams in 3-space which generate ambient isotopies for surface-links in 4-space. They are generalization of three types of Reidemeister moves for link diagrams in 2-plane which generate ambient isotopies for links in 3-space. It is known that all oriented Reidemeister moves are distinguished into different 16 versions, and then Polyak showed that all of them are generated by sets containing four or five of them. I verified that all oriented Roseman moves are distinguished into different 50 versions and proved that all of them are generated by a set containing ten of them.

### (3) Quandle cocycle invariants of immersed surface-links

An *immersed surface-link* is a closed surface generically-immersed in $\mathbb{R}^4$. (A usual surface-link is called an *embedded surface-link* to distinguish from it.) For a diagram of an embedded surface-link, one can calculate a state sum associated with a 3-cocycle of a usual quandle homology. It is known that such a state sum is an invariant of an embedded surface-link, which is called the *quandle cocycle invariant*. To introduce the quandle cocycle invariant for an immersed surface-link, I construct a new variation of a quandle homology. For a diagram of an immersed surface-link, a state sum associated with a quandle 3-cocycle in a usual sense does not become an invariant of an immersed surface-link. However, a state sum associated with a quandle 3-cocycle in a new sense becomes an invariant of an immersed surface-link.

### (4) The triple point number of an immersed surface-knot

The *triple point number* of an immersed surface-knot $F$ is defined by the minimum number of triple points required for a diagram of $F$. Professor Satoh showed that there does not exist an embedded sphere-knot whose triple point number is one, two or three. I prove that for an immersed sphere-knot with at most one self-intersection point, a similar result holds.

### (5) A simple calculation of the Arf invariant via a bicolored diagram

The Arf invariant in knot theory is a cobordism invariant defined for a knot or proper link. It is known that there are several ways to calculate the Arf invariant, e.g. using a $\mathbb{Z}_2$-Seifert form, the polynomial invariants, local moves, and 4-dimensional techniques. I introduced the notion of bicolored diagrams which is equivalent to the local move called a region crossing change and developed a new way to calculate the Arf invariant via a bicolored diagram.