(2) Study proposal Makiko Mase

(i) Aim of the study

The aim of the study is to understand algebro-geometric aspects of algebraic complex $K3$ surfaces, which we simply call a $K3$ surface . It is known that geometry of $K3$ surfaces is related to singularity theory, symplectic geometry, and the mathematical physics. It is classically an important and difficult problem to observe algebraic curves that admit a given Weierstrass semigroup: in particular, curves on $K3$ surfaces as subvarieties. In mathematical physics, it is a long-term question to characterize $K3$ surfaces from a view-point of moduli spaces of maps from a $K3$ surface to a Lie algebra. As an interesting example, for families of weighted $K3$ surfaces, it is important to describe the degenerations and compactification of the moduli space.

Problems

1. The relation between algebro-topological properties of singularities and $K3$ surfaces.
2. The moduli space of maps from a $K3$ surface to a Lie algebra.
3. Weierstrass semi-groups of pointed curves in a $K3$ surface.
4. A degeneration of weighted $K3$ surfaces in a family and the compcatification of the moduli.

(ii) Study methods

Problem 1 This is a joint work with Professor Claus Hertling in the University of Mannheim.

The deformation of a singularity produces the Milnor lattice together with the monodromy action on it. We gave a sufficient condition for the Milnor lattice together with the monodromy to admit a standard decomposition into Orlik blocks for certain singularities. We are interested in a Torelli-type theorem for the Milnor lattice with Seifert form in terms of Brieskorn lattice, which will lead to understand the Frobenius structure of the base space of unfolding for singularities. In the special case of dimension 3, we would like to apply the study to characterize $K3$ surfaces.

Problem 2 The moduli of maps are studied also in a context of singularity theory. We are interested in maps (might be with some assumption such as holomorphic) from a $K3$ surface to a Lie algebra. It is important to study first the parameter space of the moduli, for which we expect to be a set of lattices, and second the singularities of such maps.

Problem 3 This is a joint work with Professor Jiryo Komeda in Kanagawa Institute of Technology.

An interesting construction of a $K3$ surface is via a double covering of a rational elliptic surface or of a del Pezzo surface. Any rational elliptic surface is obtained by blowing up nine points in the projective plane, and all the rational elliptic surfaces are classified by describing their singular fibres. For del Pezzo surfaces, the moduli space has a strong connection with the 3-dimensional sphere, which we expect to be applied deeper study of algebraic curves with Weierstrass points on a $K3$ surface thus obtained.

Problem 4 There are many ways to describe degenerations of algebraic surfaces in a family connected to one of major problems in algebraic geometry to compactify the moduli spaces. In the study, we would like to describe some nice degeneration of $K3$ surfaces in the weighted projective spaces, where $K3$ surfaces are obtained as anticanonical sections of the space.

(iii) Aspects

As singularities give high-resolution panorama for many geometrical objects, we expect the same for our $K3$ surfaces. Moreover, by looking at relations with Lie algebras, such view shall be wider as we prospect other interpretation via the theory of Lie algebras. That can be applied to construct a moduli space together with its compactification with a relation of some deformation of singularities in the ambient space.