

Research Plan --- Yasuo Matsushita

For 2021 Academic Year

March 8, 2021

My basic and main research is again concerning the geometry on the indefinite metric spaces.

The Research Plan:

The counterexamples of the Goldberg Conjecture for pseudo-Riemannian manifolds in dimensions 6 and 8 have been reported from 2007 until 2017. For manifolds of the lowest dimensional case, dimension 4, the situations concerning the Goldberg Conjecture it are is quite different from the manifolds of higher dimensional cases. Therefore, our research target will be focused on the 4-dimensional manifolds, if the Goldberg Conjecture is true or not.

There may be a possibility to find counterexample to the Goldberg Conjecture for arbitrary even dimensions $2n$ and for arbitrary even signatures $(2n-2q, 2q)$. Therefore, it will be planned to complete calculations, and to start to write preprints concerning these issue.

1. There are several canonical types of Weyl curvatures for indefinite metric spaces, with structure groups $SO(p, q)$. If we consider the geometry of such indefinite metric spaces with their corresponding spinor groups $Spin(p, q)$, then the Weyl curvatures can be classified into further finer canonical types.
2. This approach can be highly expected to analyze the indefinite manifolds with $Spin(p, q)$ deeper than with structure groups $SO(p, q)$.
3. We already published three papers in this direction of spinor approach to neutral geometry, especially to Walker 4-manifolds. These papers are written together with Peter R. Law, a PhD student of Roger Penrose in 1983.
4. Walker manifolds are some specific manifolds, which carry a field of parallel null planes. There are, in fact, two kinds of Walker metrics, Types I and II. There are many papers on Type I, but not on Type II. Even though our analysis on Type II is still under primitive stage, we feel a large possibility to approach on Type II, which exhibit quite different aspect than Type I. This analysis can be submitted for publication soon.
5. This may become an initiation to analyze Walker Type II geometry.
6. We will be seeking if the Goldberg conjecture is true or not for four-dimensional neutral manifolds.

Above mentioned are the works to be analyzed immediately. If we consider a Lorentz metric as an operator on a manifold, then it can be a wave operator. Similarly, a neutral metric on a 4-manifold corresponds to an ultrahyperbolic operator, intensively analyzed by Fritz John. It is hoped that relations between the solutions of these second order partial differential equations (PDE's) and the curvature tensors of indefinite metric spaces, from a point of view of Physics.