Research Plan

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1. Weakly reflective submanifolds in symmetric spaces

Weakly reflective submanifolds, introduced by Ikawa, Sakai and Tasaki, are minimal submanifolds of Riemannian manifolds which have certain symmetries and are known as interesting geometrical objects. Several kinds of examples are given as orbits of isometric actions of Lie groups. However many properties of them have not been well-understood yet. In my research it was shown that a weakly reflective submanifold in a compact symmetric space gives rise to a weakly reflective PF submanifold in a Hilbert space (List [2]). Although so obtained weakly reflective PF submanifolds are infinite dimensional, the group of isometries is expected to have a simple structure due to linearity of Hilbert spaces. The purpose of this research is to study the structure of weakly reflective submanifolds via weakly reflective PF submanifolds in Hilbert spaces. This research is planed to be conducted in cooperation with Professor Takashi Sakai (Tokyo Metropolitan University).

2. Geometry of orbits of $P(G, H)$-actions induced by Hermann actions (In preparation [1])

Hermann actions constitute an important class of hyperpolar actions on compact symmetric spaces. They are also important since they induce the $P(G, H)$-action (a path group action on a Hilbert space) which are equivalent to isotropy representations of affine Kac-Moody symmetric spaces. In this research I will formulate the geometry of orbits of Hermann actions and study the geometry of $P(G, H)$-actions. In particular I compute the principal curvatures of $P(G, H)$-orbits and determine the austere $P(G, H)$-orbits. Moreover I will extend the theory of austere submanifolds in Euclidean spaces to a class of PF submanifolds in Hilbert spaces, and apply this theory to the austere orbits of $P(G, H)$-actions. Using this result I will establish a new tool of studying orbits in Hermann actions.

3. Isotropy representations of affine Kac-Moody symmetric spaces

Affine Kac-Moody symmetric spaces are infinite dimensional symmetric spaces proposed by Terng and established by Heintze, Popescu and Freyn based on Kac-Moody theory. Many similar properties between those spaces and finite dimensional Riemannian symmetric spaces are known. In particular their isotropy representations are given by hyperpolar $P(G, H)$-actions mentioned above. In the finite dimensional case it is known that there exists a unique minimal orbit in each strata of the stratification of orbit types (Hirohashi-Song-Takagi-Tasaki 2000). I conjecture that the similar property also holds for affine Kac-Moody symmetric spaces. In this research first I prove conjecture. Then I determine the symmetries of minimal orbits, compare them with the result in finite dimensional case (Ikawa-Sakai-Tasaki 2009) and clarify the similarities or differences between finite and infinite dimensional cases.

4. Reformulation of soliton theories by affine Kac-Moody groups

According to the research by M. Kashiwara, M. Jimbo, E. Date and T. Miwa, the symmetries of soliton equations are described in terms of affine Kac-Moody algebras. On the other hand, Terng and Uhlenbeck showed that transformations on solution spaces of some integrable equations can be described by loop group actions. These two researches have been conducted independently and their relation seems to be not clear. An *affine Kac-Moody group* is a group corresponding to an affine Kac-Moody algebra and recently its realization is established in the framework of affine Kac-Moody symmetric spaces. An affine Kac-Moody group is realized as a T^2-bundle over a twisted loop group of smooth loops, which has a structure of tame Frechet manifold (Hamilton 1982), and seems to be useful to study the connection between those two soliton theories. I aim to integrate the above two methods for soliton theories by affine Kac-Moody groups.