Research results achieved up to now   Name: Hideto NAKASHIMA

The following is my research results. The numbers of papers are followed by the list of my papers.

(1) **Algebraic study on homogeneous open convex cones.** Homogeneous open convex cones (we say homogeneous cones for short) are a generalization of open convex cones of positive definite symmetric matrices in a view of homogeneity. They are one of the constructing data of homogeneous Siegel domains, which form an important class in homogeneous convex domains. Basic relative invariants associated with homogeneous cones are a system of irreducible polynomials which generalize a system of principal minors of symmetric matrices. Any homogeneous cone can be described as a positive domain with respect to the associated basic relative invariants (Ishi, 2001) so that they play an important role in the study of homogeneous cones. In my paper [7], I gave an explicit and closed formula of basic relative inariants of general homogeneous cones. More precisely, I constructed an algorithm which determines the corresponding characters of the basic relative invariants in terms of structural constants of homogeneous cones, and then I gave an explicit formula of basic relative invariants by using Vinberg polynomials. Here, Vinberg polynomials, which is introduced by Vinberg(1963), are a system of polynomials, not necessarily irreducible, obtained from structural information of homogeneous cones. Until then, it is only known that basic relative invariants are obtained inductively by factoring out from a system of Vinberg polynomials (Ishi, 2001). As an application, I gave three characterizations of symmetric cones among irreducible homogeneous cones in my papers [3, 5] as follows.

1. The associated characters of the basic relative invariants correspond to the Cartan matrix of type A;
2. In the tube domains over homogeneous cones, basic relatively invariants admit a kind of positivity;
3. There exists a relative invariant polynomial such that its Laplace transform admits a kind of polynomiality.

In particular, the third one is an interesting result because it connects symmetricity of domains with polynomiality of relatively invariants functions.

(2) Studies on zeta functions associated with homogeneous cones. Associated with homogeneous cones, there exist solvable prehomogeneous vector spaces so that we can consider the associated multivariate zeta functions. In my paper [2], I considered $\mathbb{Q}$-structures of homogeneous cones which are needed to define the corresponding zeta functions. I gave a condition that homogeneous cones admit $\mathbb{Q}$-structures in terms of weight spaces of the corresponding Lie algebras, and also constructed an example of homogeneous cones which do not admit any $\mathbb{Q}$-structure. Moreover, I gave an explicit formula of their functional equation by using structural constants of homogeneous cones. To do so, I reformulated the calculation for zeta functions associated with symmetric cones given by Satake–Faraut (1984) in terms of homogeneous cones. I also obtained the explicit formula of multivariate b-functions, which control non-trivial poles of the zeta functions. In study of non-reductive prehomogeneous vector spaces, there are few calculations which determine functional equations of the associated zeta functions systematically.

(3) Researches on random matrices related to homogeneous cones. Homogeneous cones are generalizations of the convex cones of positive definite symmetric matrices on which Wishart matrices are defined so that we can expect to generalize the theory of

Wishart matrices to the theory on homogeneous cones. In our study [11] which is a joint work with Piotr Graczyk (Université d'Angers, France), we proposed a definition of Wishart matrices on a certain class of homogeneous cones which we can calculate the limiting eigenvalue distributions explicitly by means of a generalization of the Lambert function. In our paper [9] in preparation, we investigate the analytic properties of the functions in detail. This research relates to the problem of eigenvalue distributions on matrix spaces with respect to graphical models.

(4) Studies on rings of invariant differential operators on homogeneous cones. In my paper [1], I gave an explicit formula of generators of the rings of invariant differential operators on homogeneous cones and determined multiplication rules among them where we regard homogeneous cones as the homogeneous spaces on which split solvable Lie groups act simply transitively. In particular, I found that the Euler operator is always included in the center of the rings, and in a certain series of non-symmetric homogeneous cones, we have a kind of Capelli-type identity.