## Plan of Research

Shin'ya Okazaki

## Litherland's Alexander polynomial for handlebody-knots

A genus g handlebody-knot H is a genus g handlebody embedded in the 3-sphere. The Alexander polynomial is an invariant of a pair consisting of handlebody-knot and its meridian system. Replacing a meridian system of a handlebody-knot acts on the Alexander invariant as an action of  $GL(g, \mathbb{Z})$ . We introduced an invariant  $G_H$  for handlebody-knots which does not depend on the choice of the meridian system of H by using an invariant of the action of  $GL(g, \mathbb{Z})$  from the Alexander polynomial.

R. Litherland introduced the Alexander polynomial for  $\theta_g$ -curves. In general, the elementary ideal of the Alexander invariant is not principal for  $\theta_g$ -curves. Thus, there are infinitely many  $\theta_g$ -curves whose Alexander invariant is non-trivial and Alexander polynomial is trivial. However, the elementary ideal of Litherland's Alexander invariant is principal, and Litherland's Alexander polynomial for  $\theta_g$ -curve.

We extended Litherland's Alexander polynomial of a  $\theta_g$ -curve to that a pair of H and its meridian system with base point and understood how act replacing a meridian system for Litherland's Alexander polynomial of handlebody-knot  $4_1$ . We would like to consider that how act replacing a meridian system for Litherland's Alexander polynomial of other handlebodyknots.

## Torsion invariant for handlebody-knots

I would like to introduce the torsion invaeriant for pairs consisting of the exterior of handlebody-knot and its subspace by using Litherland's method. The torsion invariant is weaker than Litherland's Alexander polynomial as an invariant of handlebody-knot. However, the torsion invariant is easier than Litherland's Alexander polynomial to calculate.

## Twisted Alexander polynomial for handlebody-knots

We have some property of irreducibility of H and constituent link of H by using the Alexander polynomial as previous research. I would like to expand this result for the twisted Alexander polynomial for a handlebody-knot.