Results of my research

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A genus 2 handlebody-knot is a genus 2 handlebody embedded in the 3-sphere, denoted by H. Two handlebody-knots are equivalent if one can be transformed into the other by an isotopy of S^3. Cutting along a meridian disk system of H, if we have a knotted solid torus in S^3, then we call the spine of the knotted solid torus a constituent knot of H. The constituent knot depend on choice of a meridian disk. There are infinite many meridian disks for a handlebody-knot. Thus, there are infinite many constituent knots for a handlebody-knot. Then, we have the following results.

Let K be a knot in S^3. Let $\Delta_K(t)$ be the Alexander polynomial of a knot K. The Nakanishi index $m(K)$ of K is the minimum size among all square Alexander matrices of K.

Theorem 1 [O.]
If $K \in CK(4_1)$, then $m(K) \leq 1$ or $\Delta_K(t)$ is reducible.

Here, 4_1 is the handlebody-knot in the table of genus 2 handlebody-knots with up to six crossings. We have that the knot 9_{35} is not a constituent knot of the handlebody-knot 4_1 by Theorem 1.

Let $Conj(G_1, G_2)$ be the set of conjugacy classes of homomorphisms from a group G_1 to a group G_2. Let $G(K)$ be a knot group of K.

Theorem 2 [O.]
If $K \in CK(4_1)$, then $\#Conj(G(K), SL(2, \mathbb{Z}_3)) \leq 11$ or $\Delta_K(t)$ is reducible.

We have that the knot 10_{115} is not a constituent knot of the handlebody-knot 4_1 by Theorem 2.

Cutting along a meridian disk system of H, we have knotted solid tori in S^3. then we call the spine of the knotted solid tori a constituent link of H. Then, we have the following result.

Theorem 3 [O.]
If L is not in $\{0^2_1, 7^2_2, 9^2_6, 9^2_{23}, 9^2_{39}, 9^2_{54}\}$ and the crossing number of L is less than or equal to 9, then L is not a constituent link of the handlebody-knot 4_1.

Here, $7^2_2, 9^2_6, 9^2_{23}, 9^2_{39}$ and 9^2_{54} are 2-component links in the table of links with up to nine crossings by Rolfsen, and 0^2_1 is the two component trivial link.