## Plan of Research

We have been studying matrix models for some years, keeping in mind the "gauge theory/matrix model correspondence". We have been mainly studying the properties of the matrix models corresponding to the four dimensional  $\mathcal{N} = 2$  supersymmetric gauge theories. We would like to continue to pursue this topic.

Recently, we have studied the relation between the Painlevé system and a matrix model closely related to the supersymmetric SU(2) gauge theory with  $N_f = 2$ . The matrix model we have studied is the unitary matrix model, which is an extension of the Gross-Witten-Wadia model with a logarithmic potential. The partition function of the unitary matrix model is regarded as the tau function of the Painlevé III equation, and the relation between the partition function of the unitary matrix model and the instanton partition function of the gauge theory is investigated.

First, we will study the matrix models which correspond to the SU(2) gauge theory with general  $N_f$ . Then, we try to extend the consideration of the gauge theory/matrix model correspondence to the more general case from the perspective of the Painlevé system and its tau function. Alternatively, we would like to investigate whether it is possible to reconsider the "gauge theory/matrix model correspondence" from the viewpoint of more general integrable hierarchies and their tau functions.

We also consider whether it is possible to q-deform these matrix models. The q-deformed models are expected to be related to five-dimensional gauge theories and six-dimensional theories, therefore analyzing their properties will be an interesting research topic. These theories are also expected to be related to the q-deformed two-dimensional field theories, whose symmetry algebras form the q-Virasoro/q-W algebras.

Previously, we have studied the root of unity limit of q in two dimensional models and showed that the parafermion algebra can be obtained from the q-Virasoro/q-W algebras. By studying similar limits in detail, we would like to clarify various properties of the "gauge theory/matrix model correspondence". One of the research topics that we would like to try is how the limit is extended in the case of gauge theories on ALE spaces other than A-type gauge theories and the quiver gauge theories. A "Yangian algebra" associated with the quiver gauge theories has been proposed. In the Schur-Weyl correspondence, the Yangian algebra is related to the Hecke algebra, so an extension in this direction will deepen our understanding of the gauge theory/conformal field theory/matrix model correspondence.