Results of research

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We have been studying the properties of the matrix models which are closely related to the supersymmetric gauge theories and obtained the following results.

1. The (W)AGT conjecture implies there is a correspondence between the partition functions of the four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories and the conformal blocks of the two-dimensional theories with the Virasoro or W symmetries. The "q-deformed" version of (W)AGT conjecture states that the q-lifted version of the partition function of five-dimensional gauge theories and the "conformal blocks" of the two-dimensional theories with the q-deformed Virasoro/W symmetries. Starting from this q-version of (W)AGT conjecture, we demonstrate by taking a certain r-th root of unity limit in q, the correspondence between the four-dimensional partition function on the ALE space of A-type and the conformal blocks of the two-dimensional theories with the super-Virasoro symmetry or its generalization is automatically generated ([32]). Furthermore, we demonstrated that the parafermions appear in the r-th root of limit of the q-deformed Virasoro and the q-deformed W algebra (Ref. [34] of the Publication List).

2. A q-deformed vertex operator is determined from the five-dimensional $SU(2)$ Nekrasov partition function based on the q-AGT conjecture. We obtained a q-deformed version of Coulomb gas representation of the conformal block by using the vertex operators and q-screening charges. After slightly changing the position of one of the vertex operators, we have checked that the q-block coincides with 5D Nekrasov function in low degrees of instanton expansion ([35]).

3. A set of Schwinger-Dyson equations for the resolvents are considered in a class of supersymmetric Chern-Simons-matter matrix models. In the planar limit, these loop equations reduce to two independent algebraic cubic equations for the two planar resolvents ([36]).

4. We argue that the level-1 elliptic algebra $U_{q,p}(\widehat{\mathfrak{g}})$ is a dynamical symmetry in the correspondence between 2d field theories and 5d supersymmetric gauge theories. A level-1 $U_{q,p}(\widehat{\mathfrak{sl}}(2))$ module can be realized by an elliptic version of the Frenkel-Kac construction. In a r-th root of unity limit of the deformation parameter p, the $\mathbb{Z}_r$-parafermions and a free boson appear. And the 2d/5d correspondence goes to the correspondence between the 2d coset CFT with para-Virasoro symmetry and 4d $\mathcal{N} = 2$ $SU(2)$ gauge theory on $\mathbb{R}^4/\mathbb{Z}_r$.

5. We have argued that a unitary matrix model, which is an extension of the Gross-Witten-Wadia model by the logarithmic potential, is closely related to the $\mathcal{N} = 2$ supersymmetric $SU(2)$ gauge theory with two matter hypermultiplets. The spectral curve of the matrix model is isomorphic to the Seiberg-Witten curve of the gauge theory. Using the method of orthogonal polynomials, it is shown that the partition function is a tau function of the Painlevé equation. The double scaling limit of the matrix model corresponds to the limit of the gauge theory to the Argyres-Douglas superconformal fixed point ([38, 39]).

6. We have generalized the above gauge theory/matrix model correspondence to more general case ([41]). It is the correspondence between certain unitary matrix model and the four-dimensional $\mathcal{N} = 2$ gauge theory called the $\hat{A}_{2k,2k}$ theory. We have shown that this unitary matrix model at the k-th multicritical point corresponds to the (A_1, A_{4k-1}) Argyres-Douglas point of the $\hat{A}_{2k,2k}$ theory.