Research plan

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1. To solve a remaining problem in the paper [6]

The author in [6] gave almost complete answer to an open problem mentioned by Horiuchi and Kumlin in 2012 about existence and non-existence of minimizer of minimization problems related to embeddings from the critical Sobolev spaces to the Lorentz-Zygmund spaces. Concretely, the minimization problems have a parameter, and there exists a threshold which separates the existence and the non-existence of the minimizer. However, in the case where the parameter is equal to the threshold, it is quite delicate and difficult to show the existence or non-existence of the minimizer. It is an unknown problem. Therefore, I am going to solve it by explicit form of the potential function, rearrangement technique, and an estimate of the threshold.

2. To generalize the result in [6] to the weighted critical Sobolev spaces

The author in [6] studied minimization problems related to embeddings of the usual critical Sobolev spaces. However, the weighted case is unknown and was also mentioned by Horiuchi and Kumlin in 2012. In the weighted case, we do not apply the same method as it in [6]. Therefore, we study the weighted case by improving methods used in [6]. Concretely, we will use a weighted rearrangement technique and moving plane method in order to reduce radial setting. Furthermore, by adding a weight, two non-compactness appear and two critical exponents appear, we call this situation as “double critical case”. The double critical case is quite complicated. First, we consider the case where the minimizing sequence is concentrating at one point. Next, we consider the case where it is concentration at multi points. This will be a joint work with Professor G. Hwang (Yeungnam Univ.).

3. To generalize the result in [1] to the non-radial setting

In the paper [1], we give the explicit best constant of the sharp critical Rellich inequality and the attainability for radial functions. Since our potential function is not radially decreasing, we can not apply the higher order rearrangement technique (Talenti’s comparison principle). However, for the Hardy type inequalities, the symmetry breaking phenomenon does not occur. Therefore we can expect that our results in [1] can be extended to the non-radial setting.