今後の研究計画の英訳

佐藤 敬志

(1) Pseudo-reflection groups

Nontrivial automorphisms on a complex vector space which fix a hyperplane pointwisely is called pseudo-reflections. In the real case, the degree of such automorphism is always 2 and called a reflection simply, but those of pseudo-reflections may be more than 2. A finite group is called a pseudo-reflection group when it is generated by pseudo-reflections. Pseudo-reflection groups are natural generalization of Weyl groups, so their combinatorics and geometry must relate each other. So I hope to reveal this relation.

The classification of pseudo-reflection groups is completed by Shephard and Todd more than 60 years ago, however its combinatorics is almost not known. Keywords to understand Weyl groups are a length function and a poset structure, however they are not known for pseudo-reflection groups. Under some completion, there are analogues of flag varieties for pseudo-reflection groups, however combinatorial structure such as Bruhat decomposition is not known. My knowledge on the relation among the Weyl group, the cohomology ring of the flag variety, and the Bruhat decomposition can help me to find these combinatorial structures. Concretely, I will try to construct geometrical object as something like a cell complex by calculating the attaching map from the GKM-theoretical description of the "equivariant cohomology ring" corresponding a pseudo-reflection group.

(2)Regular semisimple Hessenberg varieties

A regular semisimple Hessenberg variety is closed under the maximal torus action, and the tangent space at some T-fixed point is the subspace corresponding to the lower ideal. A regular semisimple Hessenberg variety has a paving rather than a cell decomposition, and its equivariant cohomology ring also has a GKM-theoretical description, but it is difficult to obtain an explicit description as a quotient ring of a polynomial ring. The cohomology classes corresponding to each cells is not known. I want to give an explicit description of the cohomology ring of a regular semisimple Hessenberg variety in terms of a lower ideal and an description of the cohomology classes corresponding to each cells.

For a regular semisimple Hessenberg variety of type A, there is a special manifold which is called its Hessenberg twin. They are born like twins from a subspace of $U(n)$. Their difference comes from the left and right multiplication of a maximal torus of $U(n)$. So their torus equivariant cohomology rings are isomorphic. The complex coefficient cohomology ring of a Hessenberg twin has a different action of symmetric group from the usual one, and as a non-graded representation it is a regular representation. To determine how many times each irreducible representation appear in which degree is an interesting problem, and it was solved in the case of a flag manifold. Some combinatorial numbers of Young tableaux give the answer. I think that this number must be interpreted as counting of roots of the Weyl group of type A and that this counting with the lower ideal gives the answer in the case of Hessenberg twins. Actually I verified it works well in low dimensions.