

# Plan

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The followings are the plan of study related to the Chern-Simons perturbation theory.

1. I will revisit to the construction of the higher degree term of the Bott-Cattaneo Chern-Simons perturbation theory to remove the gap by using the technic I developed in [1] with A. S. Cattaneo.
2. I will study the  $U(1)$ (or  $SO(2)$ )-Chern-Simons perturbation theory defined by using a similar way as the Bott-Cattaneo construction. In particular, I will compare it with Lescop's invariant and Watanabe's invariant of 3-manifolds with  $b_1=1$ .
3. By using the Chern-Simons perturbation theory, we can define an invariant of a representation of a fundamental group of a 3-manifold. I will investigate the relationship between the invariant and the Reidemeister torsion of representations of the fundamental group of a 3-manifold. It is expected that this relationship gives us a geometrical (or topological) description of the Reidemeister torsion.
4. I will investigate a behaviour of the Chern-Simons perturbation theory under the appropriate surgeries among 3-manifolds with local systems (surgery formulas).
5. I will study the relationship between  $SU(2)$ -Chern-Simons perturbation theory and the Casson invariant. The Casson invariant is given as an algebraic number of irreducible representations of the fundamental group. On the other hand, for each irreducible representations, the  $SU(2)$ -Chern-Simons perturbation theory at the corresponding local system of it gives a real value. So it is expected that  $SU(2)$ -Chern-Simons perturbation theory gives a refinement of the Casson invariant.
6. I will study the Casson invariant and the Chern-Simons perturbation theory from the point of view of the singularity theory of stable maps.

## References

- [1] Alberto S. Cattaneo and Tatsuro Shimizu, A note on the  $\Theta$ -invariant of 3-manifolds, arXiv:1903.04386, 2019