Research Result

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「Keyword:Applied Mathematics(Asymptotic Expansions, Non-Linear PDE, Combustion Theory)」

My recent research topics are the propagation velocity, temperature distribution and linear stability analysis, such as Darrieus-Landau and Rayleigh-Taylor Instabilities, of a flame front, or deflagration wave. We are interested in flame fronts observed in a confined space, such as an engine of rocket and mines, or in the process of supernova explosions. Especially, the structure of Type Ia Supernova (SNIa) is well described by the mathematical model composed of non-linear partial differential equations, which targets the terrestrial combustion of oxygen and carbon. The common property between numerical and asymptotic studies of the model is an acceleration of a flame front, which in turn grows to a detonation wave and the flow transits from laminar to turbulent. This transition is welcome to enhance the efficiency of combustion, but it also has a risk of broking the machine handling the combustion phenomena. Therefore, it is desired to reveal the mechanism of the transition in view of a risk management in industry. As a first step, we focus on the initial stage of the transition and work on the mathematical analysis of a flame front propagating with small finite Mach numbers.

In the recent work[K.Wada,Combust.Theory Model.(2021)], we achieved to describe the non-adiabatic temperature distribution of a flame front propagating with small Mach numbers by use of a Lambert W function. Such a temperature profile in the SNIa was reported in previous numerical works[S.I.Glazyrin et al.(2013), D.M.Townsley et al.(2016)], but its mechanism was unrevealed theoretically. The reason for this is the assumption of the zero gradient of temperature on the burned-side edge of a flame front. This assumption is valid for the case of gas burner combustion, where the Mach number Ma, which is defined by the ratio of laminar flame speed to adiabatic sound speed, is properly approximated to be zero. However, Ma=0.01-0.1 in the SNIa, and it is impossible to neglect the change of temperature due to the effect of Mach number. For the sake of incorporating this effect, we introduced the compression zone, whose length scale is $O(Ma^2)$, and enabled the asymptotic analysis without the assumption of zero temperature gradient.

Moreover, under the large-activation-energy asymptotics, we removed the ignition temperature approximation. Then, for any temperature gradient, the burning-rate eigenvalue problem was tackled and, as a result, the formula of laminar flame speed coupled with differential equation for temperature was obtained [K.Wada(preprint)].

The effect of small heat release and viscosity for the stability boundary of thermal-diffusive instability was analyzed [K.Wada,Sci.Rep.(2021)]. Then, for small wavenumbers, the third-order term with respect to wavenumber was found to exist in dispersion relation.