(英訳)

I will continue to study quotient singularities in positive characteristic. Especially, I will consider the following problems.

1. When is a quotient singularity log terminal?

In characteristic zero, it is known that all quotient singularities are log terminal. In positive characteristic, there are quotient singularities that are not log terminal. It is a very important problem to consider the quotient singularity to be log terminal. If the order of the acting group is not divided by characteristic then the quotient singularity is log terminal as well as in characteristic zero. But, when the order is divided by characteristic, various pathological quotient singularities appear. As the simplest case, we can consider the case that the p-cyclic group acts in characteristic p. About this case, an invariant determining the class of quotient singularity is given in Yasuda(2018). As an expansion of this result, the case that the cyclic group whose order is a power of p is studied in Tanno-Yasuda(2019). In particular, the class of the quotient singularities for the representation of $\mathbb{Z}/p^2\mathbb{Z}$ are determined. These results are proved by computing an invariant of the quotient singularity using the wild McKay correspondence, which is shown by Yasuda. So we can see that the wild McKay correspondence is a powerful tool for studying quotient singularities. The next thing we would like to study is the class of quotient singularities for representations of groups which are direct products of several $\mathbb{Z}/p\mathbb{Z}$, but the computation of invariants using the wild McKay correspondence is expected to become more difficult as the number of generators increases. So we consider that finding a simpler way to compute is an important step in studying the quotient singularity. Now, I am studying the algorithm to compute $\boldsymbol{v}$-function for the representations of p-groups. After that, I want to know which datum determined the v-function and to give a simpler formula to compute v-function.

2. When does the quotient singularity have a crepant resolution?

In characteristic zero, the Gorenstein singularities in dimension lower than or equal to 3 have crepant resolutions. But it is not known in the higher dimensional case. The question of when does the quotient singularity have a crepant resolution is interesting not only in positive characteristic but in characteristic zero.

The results for the case of dimension zero and lower than three dimensions were initially proved by Roan, Ito, Markushevich by constructing crepant resolutions separately according to the classification of finite subgroups of special linear groups. In positive characteristic and in dimension 3, it is enough to consider in characteristic 3 from the result of Yasuda about the representation of the cyclic groups. I have shown that there exist crepant resolutions for a sequence of finite groups, and it is enough to consider the finite groups whose order is divided by 3 but not divided by 9. I want to study all such cases.

After the proof of Roan, Ito, Markushevich, the way to construct a crepant resolution using a moduli space G-Hilb by Matsumura and Bridgeland-King-Reid(2001). I would also like to verify whether it is possible to give such a unified crepant resolution in positive characteristic.

3. Generalization of kinds of McKay correspondence

In characteristic zero, Batyrev's theorem, which is the McKay correspondence about Hodge numbers, and categorical McKay correspondence as in Bridgeland-King-Reid hold for the quotient varieties having its crepant resolution. In positive characteristic, I construct counterexamples for Batyrev's theorem and it is known that categorical McKay correspondence does not hold. On the other hand, the McKay correspondence about motivic integration proved by Denef-Loeser(2002) in characteristic zero is extended in positive characteristic by Yasuda(2019). I would also like to deal with the question of whether Batyrev's theorem and the categorical McKay correspondence can also be generalized in some way.