(英訳)

I have been studying the singularities in positive characteristic. Among them, I have been studying quotient singularities, which is a fundamental class of singularities. The quotient singularities are the singularities that appear when a group acts on a variety and the variety is divided by this group action. It is known that quotient singularities behave well in characteristic zero. For example, it is known that all quotient singularities are log terminal, which is a classification used in the minimal model program.In particular, it has been shown by Roan, Ito, and Markushevich (1995-97) that the Gorenstein singularities in dimension 3 have a special type of resolution called crepant resolution. The question of when the crepant resolution exists is an open problem in the higher dimensional in characteristic zero, and it is one of the most interesting problems in positive characteristic.It is also known that Batyrev's theorem (Batyrev,1999) states that, if there is a crepant resolution for a Gorenstein singularity, then the topological Euler characteristic of the variety obtained by the crepant resolution coincides with the number of conjugate classes of the group. It was also an interesting problem that whether this theorem holds in positive characteristic by replacing the topological Euler characteristic with the Euler characteristic defined from the l-adic étale cohomology, which is an extension of the topological Euler characteristic in positive characteristic.

Singularities in the positive characteristic $p > 0$ do not behave well when the order of the acting group is divisible by p, which case is called "wild", and various pathological phenomena occur. As the studies about the wild case, there are Wood-Yasuda(2015), which studied the case that the symmetric group of degree n acts on the $2n$-dimensional affine space in the standard way, and Yasuda (2014,2018), which studied the case that the p-cyclic group acts on affine space. In the former, it is showed that the crepant resolution can be obtained by considering the Hilbert space of n-points. In the latter case, the classification of quotient singularities determined by the invariants defined from the action of the group. In particular, it can be seen that there exists the crepant resolution in dimension 3 only if $p = 3$. In both cases, the same argument as Batyrev's theorem holds.

On the other hand, I obtained an infinite sequence of crepant resolutions by studying the case that a finite group in the form of a semi-direct product of the 3-cyclic group or the symmetric group of degree 3 and an Abelian group whose order is not divided by 3 acts on the 3-dimensional Affine space in characteristic 3. I also obtained that the equality of Batyrev's theorem does not hold when the semi-direct product with the symmetric group. This result was shown by computing the quotient singularity and constructing its resolution concretely. Some of them can be proved by counting the $\mathbb{F}_q$-points using the wild McKay correspondence proved by Yasuda and using the Weil conjecture. Moreover, this computation can be seen as an example of the weighted counting of the Galois extensions of local fields. In Wood-Yasuda mentioned above, the counting of rational points gives another proof of the Sere-Bhagava's mass formula. So this computation can be regarded as another type of mass formula.

An important point of the computation of the wild McKay correspondence is to compute the $\boldsymbol{v}$-function defined for representation, which is a function from the pair of the extension of the local field and the group action to the rational numbers. However, the value of the $\boldsymbol{v}$-function is generally difficult to compute. Wood-Yasuda computed the $\boldsymbol{v}$-function for permutation representations, and Yasuda(2016) compute it for the hyperplane of the permutation representation. The $\boldsymbol{v}$-function for the representation of the cyclic group, whose order is a power of p, has been computed in Tanno-Yasuda(2019). While all the previous examples were computed only from the ramification filtration which is determined from the extension of the local field, I have constructed an example where the $\boldsymbol{v}$-function is not determined only from the ramification filtration.

It is known that all quotient singularities are log terminal in characteristic zero, while there exist quotient singularities that are not log terminal in positive characteristic. The question of when a quotient singularity is log terminal is one of the most important problems. It is known that the class of singularities can be determined by looking at the action of each element individually using the Reid–Shepherd-Barron–Tai criterion in characteristic zero. However, I constructed an example to which the same criterion cannot apply in positive characteristic. From this computation result, I have also obtained a determination method for when a quotient singularity in dimension 3 and characteristic 3 is log terminal.