Research Plan

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Building upon the results we obtained, we will consider the following problems:

1. Construction of semi-discrete CMC surfaces in Riemannian spaceforms

Research on semi-discrete CMC surfaces in Euclidean 3-space had been started by Müller, but he primarily focused on their theoretical aspects, so gave few examples of semi-discrete CMC surfaces. Recently, Wolfgang Carl derived a matrix representation for semi-discrete non-zero CMC surfaces in Euclidean 3-space, and showed that any semi-discrete CMC surfaces can be described by solving a pair of matrix-valued differential and difference equations. With Carl, applying matrix factorizing theorems, we gave a new construction of semi-discrete CMC surfaces in Euclidean 3-space ([15]). We will extend the result to semi-discrete CMC surfaces in 3-dimensional Riemannian spaceforms. Moreover, we will construct examples and analyze singularities of semi-discrete constant positive Gaussian curvature surfaces obtained by taking parallel surfaces of semi-discrete CMC surfaces.

2. Construction of discrete CMC surfaces in Lorentzian spaceforms

As an extension of the previous works in [2], [8], we will describe discrete CMC surfaces in 3-dimensional Lorentian spaceforms. Unlike the case in Euclidean 3-space, and like the case in Minkowski 3-space, because of the behavior of smooth surfaces, we can expect that discrete CMC surfaces in 3-dimensional Lorentzian spaceforms may have singularities. First we will determine a suitable pair of matrix-valued difference equations and derive an associated discrete integrable equation. Moreover, we will derive a new construction of discrete CMC surfaces in Lorentian spaceforms by applying the matrix factorizing theorems, and analyze their singularities using both the previous method in [2], [5] and the analysis of the corresponding discrete sinh-Gordon equation.

3. Research on discreteized timelike surfaces

We will establish a theory of discrete timelike surfaces in 3-dimensional spaceforms. In my recent work, we are describing a new theory of discrete timelike minimal surfaces in Minkowski 3-space ([14]). Unlike the other discrete surfaces, it was shown that any discrete timelike minimal surface could be described with unified coordinate systems. We will formulate integrable transformations for discretized timelike surfaces with this unified coordinate systems. With them, we will describe a theory of discrete timelike surfcaes in 3-dimensional Lorentzian spaceforms. In particular, we will derive a Weierstrass-type representation for discrete timelike CMC 1 surfaces in AdS 3-space and analyze their singularities.

Establishing a theory of discretized timelike surfaces is equivalent to extracting geometric objects of discretized wave equations. This work will contribute to further development of a theory of discretized nonlinear wave equations.

4. Analyzing singularities of general discretized surfaces

Singularities of discrete surfaces with Weierstrass-type representations had been analyzed in [2], [5], [10], [11]. As an extension, analysis singularities of trivalent maximal surfaces and their associated families was done in [9]. Toward a generalization of these previous works, we will first analyze singularities of associated families of discretized surfaces with Weierstrass-type representations.

Next we will consider more general discretized surfaces. We will analyze singularities of discrete and semi-discrete constant Gaussian curvature surfaces and their deformation families in Euclidean 3-space. It was shown that they are described by nonlinear discretized integrable equations. Like in [8], we will clarify correlation between behaviors of the corresponding discretized integrable equations and singularities of such discretized surfaces.