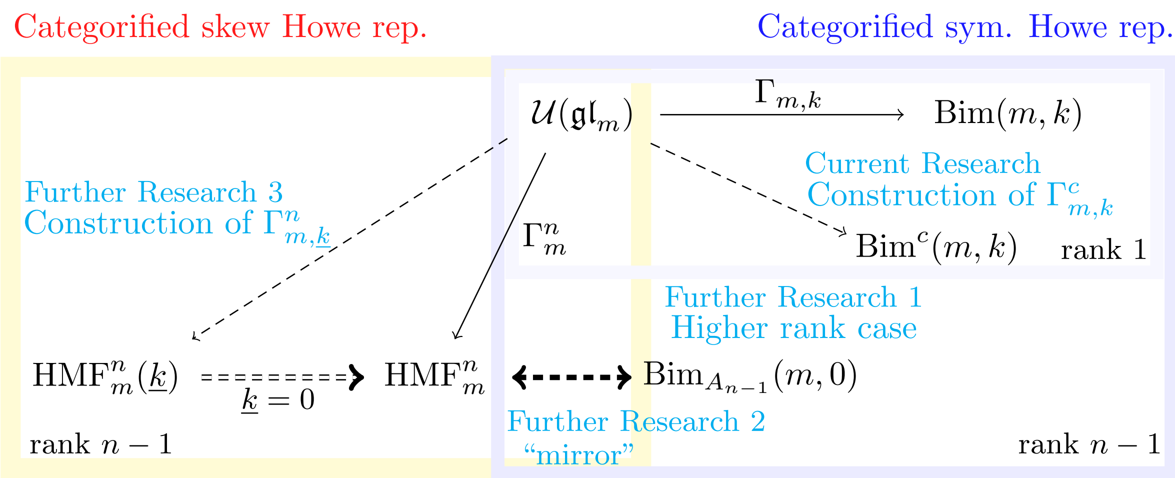

Solid arrows in the figure were results obtained in previous work. Dashed arrows indicate future areas of study.

Current Research: Currently, I'm interested in the cyclotomic quotient $W^c(\mathbf{s},k)$ of the algebra $W(\mathbf{s},k)$ and its bimodule category and I have been working on the following projects.

1. It is expected that the cyclotomic quotient is cellular. (Are these algebras isomorphic by taking a coefficient expansion of the cyclotomic quotient?).

2. Kang-Kashiwara's induction and restriction functors on $\mathcal{U}(\mathfrak{sl}_m)$ is a categorification of E_i and F_i action of $U_q(\mathfrak{sl}_m)$. It is expected that the induction and restriction functors are extended into functors on $W(\mathbf{s},k)$ and $W^c(\mathbf{s},k)$.

Further Research 1: On the symmetric product $S^k(\mathbb{C}^n\otimes\mathbb{C}^m)$, we have a left $U_q(\mathfrak{sl}_n)$ action and a right $U_q(\mathfrak{gl}_m)$ action such that these actions commute. So we have a representation

$$\gamma_m^{\mathfrak{sl}_n}: U_q(\mathfrak{gl}_m) \to \bigoplus_{\sum_{\alpha=1}^m i_\alpha=k,\ \sum_{\alpha=1}^m j_\alpha=k} \mathrm{Hom}_{U_q(\mathfrak{sl}_n)}(S^{i_1}\otimes\cdots\otimes S^{i_m}, S^{j_1}\otimes\cdots\otimes S^{j_m}).$$

It is expected that there exist a deformed Webster algebra $W^{A_{n-1}}(\mathbf{s},\underline{k})$ of type A_{n-1} and a functor from $\mathcal{U}(\mathfrak{gl}_m)$ to the bimodule category $\mathrm{Bim}_{A_{n-1}}(m,\underline{k})$. The challenge is to find "good" bimodules in the category $\mathrm{Bim}_{A_{n-1}}(m,\underline{k})$, construct concrete bimodule morphisms and then define the functor. As I mentioned above in Current Research, it is expected that the cyclotomic quotient $W^c(\mathbf{s},k)$ is a cellular algebra. The first challenge is to generalize the cellular structure into the deformed Webster algebra of type A_{n-1}.

Further Research 3: In the skew case, it is expected that there exists a category of deformed matrix factorizations $\mathrm{HMF}^n_m(\underline{k})$ with a parameter $\underline{k}\in\mathbf{Z}^{n-1}_{\geq 0}$ which is derived from the same concept as k in the algebra $W(\mathbf{s},k)$. The category $\mathrm{HMF}^n_m(\underline{k})$ in the case $\underline{k}=0$ is the category HMF^n_m. First, I'll consider the case of A_1.

Further Research 4: The p-DG structure is a generalization of DG structure whose derivation ∂ has the condition $\partial^p=0$. Using this structure, we try to define homological three-manifold invariants.