Abstract of the results of my research

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Representation theory of finite-dimensional algebras

The aim of representation theory of algebras is to investigate the structure of their module categories. So far I mainly have been studying the derived categories, the equivalences between them and some invariants under derived equivalences. They are useful tools in representation theory of algebras. For example, the important information of algebras such as the Grothendieck group and the finiteness of the global dimension is invariant under derived equivalences. Recently, it has turned out that they play important roles in Lie theory and noncommutative algebraic geometry.

The most important of various homological invariants in representation theory is the global dimension. An important concept introduced recently in connection with the global dimension is Rouquier's dimension of triangulated category. The derived dimension (=the dimension of the derived category) and the stable dimension (=the dimension of the stable derived category) of algebras are closely related to the global dimension and the representation dimension (Rouquier) so that they are especially interested in representation theory of algebras.

Therefore, I considered that the representation-theoretic properties of algebras could be controlled by the derived dimension and the stable dimension. First, I would investigate the representation-theoretic properties of self-injective algebras having low stable dimension. Indeed, I proved that it has stable dimension 0 if and only if it is representation-finite [2]. As an application, I exhibited in [4] (or [3]) that algebras having derived dimension 0 are closely related to self-injective ones having stable dimension 0. Subsequently, I focused on Iwanaga-Gorenstein(=IG) algebras, a generalization of self-injective algebras, and their derived dimensions. It is, however, still a hard problem in general to give a precise value of the dimension of a given triangulated category. We hence introduce a concept of dimension of a triangulated category with respect to a fixed subcategory in [5] (This is joint work with Aihara, Araya, Iyama, and Takahashi [5]). Our method not only recovers some known results on the derived dimensions in the sense of Rouquier, but also applies to various commutative and non-commutative noetherian rings. As a consequence, for a cotilting module T having injective dimension $d \geq 1$ over a noetherian ring Λ, we obtained the fact that Λ has derived dimension with respect to $\mathcal{X}_T$ (constructed by T) is equal to d [8].

In addition, I obtained the results related to a derived equivalence classification of self-injective algebras given by repetitive algebras with Asashiba, Kimura, and Nakashima [9].

Topological data analysis (Persistent homology)

Persistent homology, one of the main tools in topological data analysis, is used to study the persistence of topological features such as holes, voids, etc, in data. Note that the persistence diagram, which is the analysing object of persistent homology, is exactly the indecomposable decomposition of the given representation of the quiver of A type. While there is a need for practical tools applying the ideas of persistence to multiparametric data, multidimensional persistence is known to be difficult to apply in full generality because we require the indecomposable decomposition of the modules over a representation-infinite algebra. Therefore, I study solutions for such difficulties.

First, I developed a decomposition theory of modules by using Auslander-Reiten theory with Asashiba and Nakashima[6, 7]. By using this result, I discussed the interval decomposability of multidimensional persistence modules and gave a method determining the decomposability under certain finiteness conditions with Asashiba, Buchet, Escolar and Nakashima [10]. Moreover, I introduced an interval decomposable approximation and suggested a generalized persistent diagram with Asashiba, Escolar and Nakashima [11].

On the other hand, the usual persistence diagram depends on the base field of homology. I verified an algorithm determining the dependency with Obayashi [12]. In addition, I investigated distances between persistent homologies as representations [13] and a robustness of persistence diagrams by using derived equivalences [14].