RESEARCH PLAN

AIRI ASO

We would like to study following two topics.

Topic 1. To compute the limit of the complexification of Goda's formula

As in the research results, we are considering the complexification of Goda's formula [1]. First, we studied the asymptotic behavior of the twisted Alexander polynomials of hyperbolic knots up to 6 crossings associated to their $\mathrm{SL}(n,\mathbb{C})$-representations obtained from their holonomy representations. By this study, we conjectured that the real part of the formula approachs to the hyperbolic volume and the imaginary part approachs to the $2\pi^2$ times of the Chern-Simons invariant as n gets bigger. However, we have not proved that the imaginary part converges to the Chern-Simons invariant. Hence we would like to compute the limit of the formula and have following two ideas.

1. It is known that the complexification of the volume conjecture of colored Jones polynomials for knots $6_3, 8_9, 8_{20}$ are proved [2]. By using their idea, we obtain the limit our formula.
2. Ohtsuki–Takata showed a relation between the Hessian of the potential function and the Reidemeister torsion for a two-bridge knot [3]. Hence it is expected that we can obtain some relations between the twisted Alexander polynomials and the potential functions by using their arguments. Since the complex volume is given as the critical value of the potential function [4], we may obtain not only the limit value but also its generalization.

Topic 2. To obtain the holonomy representaions of Kinoshita-Terasaka knot and its generalization

Kinoshita-Terasaka knot is a 11 crossing knot with trivial Alexander polynomial, and there is a family of its generalization [5]. We would like to obtain their holonomy representations and compute the twisted Alexander polynomials associated to their $\mathrm{SL}(n,\mathbb{C})$-representations obtained from their holonomy representations. The generalization of Kinoshita-Terasaka knot is an infinite family of knots. In general, it is difficult to obtain the holonomy representations of infinite family of knots. However, it is expected that we can obtain their holonomy representations by using the similar way as in [6], since this family has knot diagrams as in the case of $(-2,3,2n+1)$-pretzel knot.

References

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