

## Research Plan

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The geometrical mapping method that we have been working on in our previous work [7] is a powerful tool for comprehensively understanding the dynamics of dynamical systems. The shape of a factor graph of mechanical word of order  $n$  with slope  $\alpha$  depends on the position of  $\alpha$  in the Farey sequence of maximal denominator  $n + 1$ , and so on. We have further shown that the position of the prefix of a mechanical word of length  $n$ , determined by the slope  $\alpha$  and the intercept  $\rho$ , and the position of the prefix in the factor graph. We will attempt to determine the relationship between the position of the prefix and  $\alpha, \rho$ .

We will also examine how far the geometric mapping method can be applied. The Sturmian words are the first step in this direction.

Since Sturmian words are closely connected to various fields of mathematics, there are multiple directions in which the research can be taken. Specifically,

- Consideration of the relationship with return words, derivated sequences, which is more general than Sturmian words [5]
- Considering of increasing the number of letters in the alphabet [2]
- Loosening the height difference condition as a variation on the concept of balance
- Loosening the periodicity condition (quasiperiodicity) [3]
- Investigation of Ostrowski numeration system [4], [6]
- Numeration systems and tilings as one extension to higher dimensions [1]

can be considered. We will explore current research for further extensions and generalizations in these related areas.

## References

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