

I studied the following topics:

- (1) L^p -Lyapunov inequalities,
- (2) Minimization problems on Hardy-Sobolev inequality,
- (3) On compactness of Sobolev embedding involving variable exponent,
- (4) Variational problem and elliptic equations on Trudinger-Moser inequality.

In these topics, I introduce (1), (2) and (4) in detail.

(1) L^p -Lyapunov inequalities

L^p -Lyapunov inequalities is a necessary condition of existence of solution on a linear elliptic equation. This problem is related to the Sobolev embedding and corresponding semilinear elliptic problem. I obtained the L^p -Lyapunov inequality in the critical case of the Neumann problem. In addition, I showed the L^p -Lyapunov inequality in the case of the nonlinear Neumann problem. This work was joint work with F. Takahashi.

(2) Minimization problems on Hardy-Sobolev inequality

I studied the Neumann problem on the Hardy-Sobolev inequality. This inequality is related to the embedding from Sobolev spaces to the Lebesgue spaces with weighted function. By singularity of the weighted function and an invariance of a scaling of functions, the embedding is non-compact. Concerning this problem, some topological properties of bounded domain plays a important role. As the previous work, it is known that there is a minimizer when the singularity is located on the boundary and the mean curvature at the singularity is positive. I studied other cases and I showed that the scale of the domain is related to the existence and nonexistence of minimizer. Under some conditions, I clarified the necessary and sufficient condition on existence of a minimizer. Based on these results, I studied the related elliptic equation with C.-H. Hsia and G. Hwang.

(4) Variational problem and elliptic equations on Trudinger-Moser inequality

It is known that there exists a extremal for the classical Trudinger-Moser inequality. By Mancini-Martinazzi (2017) , it was shown that the asymptotic expansion of the Dirichlet energy of some concentrating sequence, which is strictly related to the variational problem. After their study, P.-D. Thizy (2018) and Ibrahim-Masmoudi-Nakanishi-Sani (2020) showed the threshold nonlinearity dividing existence and nonexistence of maximizer. At the same time, I also showed one of the threshold nonlinearity dividing existence and nonexistence of a maximizer and clarified the relationship between existence of a maximizer and a behavior of lower order perturbation in detail.

In this year, I studied positive critical points of the Trudinger-Moser functional. The uniqueness result and some asymptotic properties of the critical points were proved.