

Future Research Plans

Classification of representations of non-quasi-split $SO(2n + 1)$

I am currently working on the classification of the automorphic representations and local representations of $SO(2n + 1)$. This has already been well studied by Arthur, where there is a technical assumption that $SO(2n + 1)$ is quasi-split.

As a prior study, there is a preprint of Kaletha et al., which studied the classification of the representations of general unitary groups that are not necessarily quasi-split. Referring to this, I am trying to prove the classification theorem of automorphic and local representations of non-quasi-split $SO(2n + 1)$ by regarding it as an inner form of split $SO(2n + 1)$ and reducing the problem to the results of Arthur. The classification of automorphic and local representations is a significant step in the Langlands correspondence, which is a great conjecture of number theory, and has significance in that regard. In addition, there is another important feature of this problem. Since a correspondence between the representations of $Mp(2n)$ and $SO(2n + 1)$ is given by the studies by Gan-Savin and Gan-Ichino, when I give the classification theorem of representations of non-quasi-split $SO(2n + 1)$, the classification theorem of the representations of the metaplectic group $Mp(2n)$ will be given.

This study is already expected to be completed. I am also writing a paper following this.

Newforms of half-integral weight and local new vectors of the metaplectic group $Mp(4)$

I will investigate newforms as a development of Ibukiyama's conjectures. There is an information called the level of modular forms. A modular form of smaller level is called a newform. In the case of degree 1, there is a study of newforms in the results of Shimura, given by Ueda-Yamana. Therefore, following this, I will study newforms in Ibukiyama's conjectures in the context of modular forms of half-integral weight and of degree 2. On the other hand, a local new vector is a vector in a representation of a group over a local field, which is a local component of an automorphic representation, corresponding to a newform. In this project, we investigate the existence and the number of local new vectors in the representations of $Mp(4)$ and newforms of half-integral weight and of degree 2.

First of all, for newform, the case of integral weight is well known, so we get results by constructing a similar isomorphism between them of half-integral weight and integral weight as Ibukiyama's conjectures. There is a prior study by Ueda-Yamana in the case of degree 1. Referring to this, I will make predictions, and prove it by applying the method of the proof ([Is2]) of Ibukiyama's conjectures. On the other hand, for local new vectors, I will extend a previous study in the case of the degree 1 by Roberts-Schmidt to the case of degree 2. Roberts-Schmidt's study uses Waldspurger's results to investigate the existence and the number of local new vectors of $Mp(2)$. I will utilize the results of Gan-Savin and Gan-Ichino, which are generalizations of the studies of Waldspurger. Finally, I will compare the results of researches of newforms and of local new vectors.

In the field of automorphic representations, there is an important conjecture called "GGP conjecture" about the automorphic representations, representations over local fields, and their L-functions. Newforms are useful for calculating the L-functions, and the information of the existence of newforms and local new vectors of small level has great value.