Summary of past research results

My research field is the modular forms of half-integral weight and representations of metaplectic groups.

＜Motivation and background＞

A modular form is a holomorphic function on the upper half plane with a certain symmetry. Studying the modular forms and their L-functions has long been one of the main goals of the field of number theory. There is a datum called weight attached to a modular form. In number theory, we consider the case of integral or half-integral weight. The two cases are complementary and both are important. However, the history of theory of the modular forms of half-integral weight is shorter than that of integral weight.

An automorphic representation is a representation of the group of the adelic points of an algebraic group over a number field. An automorphic representation is an abstraction of modular forms. For example, the modular forms of half-integral weight can be understood by the automorphic representations of metaplectic groups $\mathrm{Mp}(2n)$. An automorphic representation can be decomposed into the product of the representations of groups over local fields, and these global and local representations are inseparable.

In the theory of the modular forms of half-integral weight, the study of Shimura and the study of Waldspurger are typical. Shimura described the relationship between the modular forms of half-integral weight and those of integral weight. After that, Waldspurger described the correspondence between the representation of the metaplectic group $\mathrm{Mp}(2)$ and the special orthogonal group $\mathrm{SO}(3)$. Moreover, there have been two important developments in this field in recent years. One is a work of Gan-Savin, which gave a natural correspondence between the representations of the odd special orthogonal group $\mathrm{SO}(2n+1)$ and the metaplectic group $\mathrm{Mp}(2n)$ over local fields. The other is a work of Gan-Ichino, which described the correspondence between the automorphic representations of $\mathrm{Mp}(2n)$ and those of $\mathrm{SO}(2n+1)$. In other words, they proved the classification theorems of representations of $\mathrm{Mp}(2n)$ under the assumption that the same is known for non-quasi-split $\mathrm{SO}(2n+1)$.

＜My previous research＞

Local intertwining relations for $\mathrm{Mp}(2n)$

Based on the results of Gan-Savin, I have formulated the local intertwining relation for $\mathrm{Mp}(2n)$, and proven it by using the local intertwining relation for $\mathrm{SO}(2n+1)$ in a paper [Is1]. A local intertwining relation is a relation that describes how the local intertwining operators act on parabolically induced representations. Because the classification of representations is made using parabolically induced representations, local intertwining relation gives more detailed and specific classification of representations.

The results of Gan-Savin apply a correspondence that connects representations of different groups called " the theta correspondence". There is a theory called " the mixed model", which tells us the relation between the theta correspondence and the intertwining operators. I used the mixed model to solve this problem.

Ibukiyama's conjectures

In a paper [Is2], I have proven Ibukiyama's conjectures, which are important long-unresolved conjectures of half-integral weight modular forms predicted by Ibukiyama, by applying Gan-Ichino's results. The conjectures give the relationship between the modular forms of half-integral weight and those of integral weight, and can be regard as degree 2 version of Shimura's study.

The method of applying the theory of automorphic representations to the study of the modular form of integral weight has been well known. However, in this project, I found that it is very difficult to utilize the method in the case of half-integral weight, because a prime 2 is “bad”. Then I clarified that this problem can be cleared by applying the representation theory of the Jacobi group together. Of course, Ibukiyama's conjecture is valuable. This work is also valuable in the sense that it showed the method of applying the theory of Gan-Ichino to the research of the modular forms of half-integral weight.