SUMMARY OF RESEARCH

My research field is the representation theory of reductive groups defined over local fields. This gives a local aspect of the theory of automorphic forms, which are certain functions on the group of the adelic points $G(\mathbb{A}_F)$ of a reductive group G over a local field F. Especially, I have treated **classical group s** (symplectic groups, special orthogonal groups, unitary groups, quaternionic unitary groups). Here, a quaternionic unitary group is an algebraic group defined as a unitary group of (skew)-Hermitian space over a division quaternionic algebra D over F. I am interested in studying representation theoretic properties via Langlands parameters. My first and second works are on **local factors**, which are important invariants obtained from L-parameters. And my recent work is on the behavior of formal degrees under local theta correspondence, which has a background in the computations of Langlands parameters.

Local Factors. I studied local factors of irreducible representations of reductive groups defined over local fields. Here, the "local factors" consist of local L-factors, local ϵ-factors, and local γ-factors. The former two appear in the Eular product expansions of (global) L-functions and root numbers of irreducible automorphic representations. The local γ-factors are defined using the product and quotient of the L-factors and the ϵ-factors, which are expected to have good properties for parabolic inductions. In general, they are defined via the local Langlands correspondence. Let F be a local field and let G be a connected reductive group over F. **This study aims to give an analytic definition of the local factors of irreducible representations of $G(F)$ without using L-parameters**. For the merit of analytic definitions, we can relate the local factors with representation theoretic properties naturally. (One can find an example in §1.2 below.) If F has characteristic 0, then we can define the (standard) local factors of irreducible representations of non-quaternionic classical groups by using the doubling method of Piatetski-Shapiro and Rallis (see [LR05]). I have extended the result of [LR05] to

(1) the case F has characteristic 0 and G is a quaternionic unitary group ([Kak20b]), and
(2) the case F is a field of odd characteristic and G is a classical group ([Kak21]).

More precisely, for an irreducible representation π of $G(F)$, a character χ of $F^\times$, and a non-trivial additive character ψ of F, I constructed a function $\gamma(s, \pi \times \chi, \psi)$ by using the doubling method, I choose ten properties which the γ-factor is expected to satisfy, and I proved that $\gamma(s, \pi \times \chi, \psi)$ is the unique function satisfying the ten properties. Finally, we can define the L-factors and the ϵ-factors from the γ-factors conversely.

Formal Degrees and Local Theta Correspondence. Then, I studied the behavior of the **formal degrees** under the **local theta correspondence** [Kak20a]. Here the formal degree is an invariant of an irreducible square-integrable representation of $G(F)$, which is conjectured to have an explicit formula (**formal degree conjecture** [HII08]) in terms of the Langlands parameters. Moreover, the description of the local theta correspondence in terms of Langlands parameters has been established for non-quaternionic reductive dual pairs (**Prasad conjecture**), and I am studying the extension of this to quaternionic dual pairs. I obtained certain formulae as a result of this research. Although we need no hypotheses to prove the formulae, they are compatible with the above two conjectures.

The strategy of the research is the following: Let (G, H) be a quaternionic dual pair of almost equal rank. Then we can define the local theta correspondence

$$\theta_\psi(\ , H) \colon \mathrm{Irr}(G(F)) \to \mathrm{Irr}(H(F)) \cup \{0\}.$$

Let $\pi \in \mathrm{Irr}(G(F))$ be a square-integrable irreducible representation of $G(F)$ so that $\sigma = \theta_\psi(\pi, H)$ is not 0. Then one can show that σ is also square-integrable, and we can define the formal degree $\deg \sigma$. Then we define the constant $\alpha(G, H)$ so that it satisfies

$$\frac{\deg \pi}{\deg \sigma} = \alpha(G, H) \cdot c_\pi(-1) \cdot \gamma(0, \sigma \times \chi_G, \psi).$$

Here, χ_G is a character of $F^\times$ determined by G. Then, by the observation of the local analogue of the Rallis inner product formula by Gan-Ichino [GI14], we can show that $\alpha(G, H)$ does not depend on π and it is expressed by using the constant $\beta(G, H)$ appearing in the local Siegel-Weil formula. I found an integral equation ([Kak20a, Lemma 7.8]) connecting local zeta values with special values of local intertwining operators, and I computed $\beta(G, H)$. Therefore, we have obtained the description of the behavior of the formal degrees under the local theta correspondence.

References

[GI14] Gan, W.T., Ichino, A.: Formal degrees and local theta correspondence. *Invent. Math.*, 195(3):509-672, 2014.
[HII08] Hiraga, K., Ichino, A., Ikeda, T.: Formal degrees and adjoint γ-factors. *J. Amer. Math. Soc.*, 21(1):283-304, 2008.
[Kak20a] Kakuhama, H.: Formal degrees and local theta correspondence: quaternionic cases. *preprint.*, arXiv:2012.04219, 2020.
[Kak20b] Kakuhama, H.: On the local factors of irreducible representations of quaternionic unitary groups. *Manuscripta Math.*, 163(1-2):57-86, 2020.
[Kak21] Kakuhama, H.: On the local doubling γ-factor for classical groups over function fields. *J. Number Theory*, 2021.
[LR05] Lapid, E-M., Rallis, S.: On the local factors of representations of classical groups. In *Automorphic representations, L-functions and applications: progress and prospects*, volume 11 of *Ohio State Univ. Math. Res. Inst. Publ.*, 309-359. de Gruyter, Berlin, 2005.