My achievements

(I) Reserach of calibrated submanifolds

I have mainly studied the properties of explicit calibrated submanifolds. Explicit examples are very important because they provide local models of singularities and are used for the desingularization by the gluing construction. However, calibrated submanifolds are defined by nonlinear PDEs, which makes the explicit construction very difficult. In [1], I constructed examples of special Lagrangian submanifolds in a toric Calabi-Yau manifold by applying the moment map technique on $\mathbb{C}^n$ by Joyce. In [2, 9, 17], I classified homogeneous coassociative submanifolds and gave many cohomogeneity one examples in an asymptotically conical G_2-manifold by the symmetry of Lie groups. Then I could find examples with conical singularities and their desingularizati ons.

In [4, 6, 16, 7], I studied the infinitesimal (first order) deformations and the second order deformations of explicit homogeneous Cayley cones for the analysis of singularities. By the representation theory and geometric considerations, I clarified the local structures of moduli spaces of some of these cones.

(I') Introduction of affine Legendrian submanifolds

In [5], I introduced affine Legendrian submanifolds, which is a generalization of Legendrian submanifolds. I defined a new functional ϕ-volume and showed that minimal affine Legendrian submanifolds with respect to the ϕ-volume are stable under certain conditions. I defined a canonical connection on the infinite dimensional space of affine Legendrian submanifolds and defined the notion of geodesics on the space. Then I showed that the ϕ-volume is geodesically convex under certain conditions.

(II) Research of the topology and the moduli space of $G_2, \mathrm{Spin}(7)$-manifolds

In [8, 10, 18], I studied $G_2, \mathrm{Spin}(7)$-manifolds using an algebraic structure called the Frölicher Nijenhuis bracket in collaboration with H. V. Lê and L. Schwachhöfer. In [12], together with H. V. Lê, L. Schwachhöfer and D. Fiorenza, I further developed and generalized this. From the consideration of general graded differential algebras, we found new topological obstructions for a manifold to admit a metric with non-negative Ricci curvature. (Especially, these are new obstructions for a manifold to admit $G_2, \mathrm{Spin}(7)$-structures.) In [11], I found the common properties of moduli spaces of various geometric structures. As an application, I investigated the difference in the structure of the metric completions when the canonical metric in the space of the G_2 structures or the Ebin metric in the space of the Riemannian metrics is conformally transformed.

(III) Moduli theory of dHYM and $G_2, \mathrm{Spin}(7)$-dDT connections

We can consider dHYM, $G_2, \mathrm{Spin}(7)$-dDT connections as an analogue of calibrated submanifolds or HYM, G_2, Spin(7)-instantons, but it is necessary to confirm whether the similarity holds rigorously. In [13-15, 19], I showed that many similarities in the moduli spaces actually hold in collaboration with H. Yamamoto.

First, I introduced new geometric structures and showed that deformations of dHYM, $G_2, \mathrm{Spin}(7)$-dDT connections are controlled by a subcomplex of an elliptic complex. More strongly, it is shown that the connected component of the moduli space $\mathcal{M}$ is a torus if there are no obstructions of deformations. I also showed that $\mathcal{M}$ is a smooth manifold if we perturb the structure generically, and that $\mathcal{M}$ admits a canonical orientation (which is a global structure).

The above-mentioned "mirror" is obtained by the "real Fourier–Mukai transform" for a torus fibration. Lee-Leung defined a Spin(7)-dDT connection by this method, but there were problems such as the incompatibility with other geometries. I computed this transform again carefully, and suggested an alternative definition of a Spin(7)-dDT connection which seems more appropriate in [14]. I also conjectured the mirror version of Cayley and associator equalities, which are basic but important equations in $G_2, \mathrm{Spin}(7)$ geometry. In [19], I showed that it actually holds. Using this, I further show the following.

(1) $G_2, \mathrm{Spin}(7)$-dDT connections minimize the "volume" V, and its value is given topologically. (This corresponds to the fact that a calibrated submanifold is volume minimizing in its homology class.) (2) Any $G_2, \mathrm{Spin}(7)$-dDT connection on flat bundles must be flat. (3) The structure of the moduli space is determined when the holonomy group of a G_2- or Spin(7)-manifold reduces.

Here, "volume" V is a mirror of the standard volume for submanifolds. (In physics, it is called the Dirac-Born-Infeld (DBI) action.) This gradient flow (mirror mean curvature flow) is not parabolic as in the case of submanifolds. In [19], I describe the "mirror mean curvature" nicely, which enables us to apply the Deturck's trick to obtain the short-time existence and uniqueness of the flow.