Plans

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(1) Research on a minimal generating set of oriented Roseman moves

A surface-diagram is the image of a surface-knot via a generic projection from 4-space into 3-space, equipped with height information. It is known that two surface-diagrams are related by a finite sequence of seven types of Roseman moves if and only if they present the same surface-knot. When we construct an invariant of an oriented surface-knot from a surface-diagram, it is sufficient to show that the quantity does not change under seven types of Roseman moves. Since seven types of Roseman moves are divided into different 50 versions with respect to height information and orientations of surfaces, it is not easy to check the invariance under all the 50 versions of oriented Roseman moves. Therefore, it is simplified the process to show the invariance if we can find a collection of some oriented Roseman moves which generates all oriented Roseman moves. We call such a collection a generating set of oriented Roseman moves. There is a generating set of oriented Roseman moves consisting of 10 versions by my recent work. The purpose of this research is to construct a minimal generating set of oriented Roseman moves, which means a generating set whose proper subset cannot generates all the 50 versions.

(2) Research on the augmented Alexander invariant of an oriented surface-knot

Recently, Atsushi Ishii and Kanako Oshiro are defined the augmented Alexander invariant of an oriented knot, which is a generalization of the twisted Alexander invariant and the quandle cocycle invariant. In this research, collaborating with them, we develop the augmented Alexander invariant of an oriented surface-knot, and clarify relation between the augmented Alexander invariant and the quandle cocycle invariant of an oriented knot. The augmented Alexander invariant of an oriented knot is calculated by constructing a matrix from a diagram of the oriented knot. We have investigated behavior of a analogous matrix constructed from a surface-diagram of an oriented surface-knot under seven types of Roseman moves, combining with **(1)**. From now on, we define a quantity cancelling change of the matrix under seven types of Roseman moves, and develop the augmented Alexander invariant of an oriented surface-knot. Since surface-diagrams and their deformations are complicated, we consider oriented sphere-knots or surface diagrams with no branch points if necessary.

(3) Research on the quandle cocycle invariant of an oriented singular surface-knot

A quandle is an algebraic system derived form oriented Reidemeister moves, and gives us invariants of oriented knot and oriented surface-knot. One of famous invariants of an oriented surface-knot is the quandle cocycle invariant which is obtained from a 3-cocycle of a quandle, and it is applied to detection of the non-invertibility and estimation of the triple point number. It is known that the quandle cocycle invariant does not usually becomes an invariant of an oriented singular surface-knot, where a singular surface-knot is a surface-knot with self-intersection points. This is because the quandle cocycle invariant changes under the eighth type of Roseman moves related to self-intersection points. By my recent work, a quandle cohomology group can be modified so that the quandle cocycle invariant becomes an invariant of an oriented singular surface-knot. The purpose of this research is to calculate concrete examples of modified 3-cocycles of a quandle using computer, and to apply them to detection of the non-invertibility and estimation of the triple point number.