Results

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(1) Research on local property of Roseman moves and surface-diagrams

A surface-diagram is the image of a surface-knot via a generic projection from 4-space into 3-space, equipped with height information, and a method using a surface-diagram has been developed since the 1990s. Equivalence deformation of a surface-knot in 4-space is described by a surface-diagram and seven types of local transformations, which are called Roseman moves, of a surface-diagram. I proved that six types of Roseman moves can generate all seven types, but five or less types can't (refereed paper [6]).

(2) Research on global property of Roseman moves and surface-diagrams

Result (1) means that two surface-diagrams of a surface-knot are related by a finite sequence of at least six types of Roseman moves, but it does not give us an answer to the following question: for two surface-diagrams of a surface-knot, what types should appear in a sequence of Roseman moves between them? Collaborating with Kanako Oshiro and Kokoro Tanaka, we proved that there exist infinitely many pairs of surface-diagrams of a surface-knot such that any sequence of Roseman moves between them must contain moves involving triple points (refereed paper [5]).

(3) Research on ribbon-clasp surface-knots

I had tried to extend the clasp number, an invariant of a knot which I researched in my refereed paper [7], to an invariant of a surface-knot. Its attack fell through, but I found a new class of surface-knots which called a ribbon-clasp surface-knot. A ribbon-clasp surface-knot is a generalization of a ribbon surface-knot which is well-researched in surface-knot theory. Collaborating with Seiichi Kamada, we gave a geometrical characterization of a ribbon-clasp surface-knot in several ways. Moreover, we defined a normal form of a surface-knot with self-intersection points, which is called a singular surface-knot, and proved that any singular surface-knot can be deformed into a normal form (refereed paper [4]).

(4) Research on the minimum number of triple points included in surface-diagram

In knot theory, it is known that the minimum number of crossings included in a diagram of a knot is not equal to one or two. In surface-knot theory, it is known that the minimum number of triple points included in a surface-diagram of a sphere-knot (with no self-intersection points) is not equal to one, two or three. I had tried to prove a similar result for a surface-knot with self-intersection points, which is called a singular surface-knot, and obtained the following result (refereed paper [2]): The minimum number of triple points included in a surface-diagram of a singular sphere-knot with one self-intersection point is not equal to one, two or three.

(5) Research on the region crossing change and the Arf invariant

The Arf invariant in knot theory is a cobordism invariant defined for a knot or proper link. It is known that there are several ways to calculate the Arf invariant, e.g. using a $\mathbb{Z}_2$-Seifert form, the polynomial invariants, local moves, and 4-dimensional techniques. I introduced the notion of bicolored diagrams which is closely related to the region crossing change, and gave a new way to calculate the Arf invariant via a bicolored diagram (refereed paper [1]).