RESEARCH PLAN

KIMBALL MARTIN

Periods of automorphic forms are important tools to study L-functions and functorial lifts in the Langlands program. For instance, formulas for central values of L-functions in terms of periods for GL(2) have applications to subconvexity and equidistribution problems, the Bloch–Kato conjecture, and ranks of elliptic curves. In 1992, Gross and Prasad conjectured a relation between L-values and periods in a codimension 1 setting in higher rank.

To be somewhat precise, consider a codimension 1 pair $V \supset W$ of quadratic spaces. Let π and τ be cuspidal automorphic representations of $G = \mathrm{SO}(V)$ and $H = \mathrm{SO}(W)$. Then the original Gross–Prasad conjecture relates the nonvanishing of $\mathrm{SO}(W)$-periods for $\pi \times \tau$ to the nonvanishing of $L(\frac{1}{2}, \pi \times \tau)$. To be more precise, the nonvanishing a period implies the nonvanishing of an L-value, but for the converse one needs to also consider periods on relevant inner forms. In particular, periods can only be carried on a single inner form due to Gross and Prasad's local dichotomy principle. The Gan–Gross–Prasad (GGP) conjectures extend this to include the case where $V \supset W$ is of higher codimension as well as treat pairs (G, H) of other types of classical groups (e.g., unitary groups). There has been much work on establishing the GGP conjectures, and now much is proven.

Outside of the GGP conjectures, we know a number of other instances of how certain periods are, or at least should be, related to L-values and functorial lifts. For instance, Pollack, Wan and Zydor recently showed that the nonvanishing of periods over $H = \mathrm{SO}(n+1) \times \mathrm{SO}(n)$ on the quasisplit group $G = \mathrm{SO}(2n+1)$ is related to the nonvanishing of central L-values of G.

During my visit to Osaka Metropolitan University, I plan to investigate the relation between periods and L-values in new cases with Masaaki Furusawa. We will begin with the special case that G is an inner form of SO(5). Then the analogue of the above work of Pollack, Wan and Zydor is to consider periods on inner forms H of $\mathrm{SO}(3) \times \mathrm{SO}(2)$, we will attempt to compare them with central L-values on SO(5) using the theta correspondence. Understanding how these periods behave for different inner forms should explain the analogue of the Gross–Prasad dichotomy principle in this context.

One upshot of this would be a better understanding of restriction problems in local representation theory. Such a relation should also have applications to the study of L-values on SO(5). For instance, one should be able to deduce some explicit nonvanishing of central L-values for $G = \mathrm{SO}(5)$ along the lines of some of my previous for for $G = \mathrm{PGL}(2) \simeq \mathrm{SO}(3)$.