RESEARCH SUMMARY

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The main focus of my research is in number theory, automorphic forms, and representation theory. I will highlight 3 themes of my research.

1. Central L-values and periods of automorphic forms. My 2009 *IMRN* paper with Whitehouse uses the relative trace formula to establish an explicit formula for twisted GL(2) L-values in terms of toric periods, which refines a fundamental formula of Waldspurger for Gross–Prasad test vectors. This has various applications to subconvexity, averages and non-vanishing mod p of L-values, as well as to constructions of p-adic L-functions. Later I refined this work in my paper 2017 *Algebra and Number Theory* paper with File and Pitale, as well as used this formula in joint work with Longo and Hu (*Ann Math Quebec*, 2020) on studying Stark–Heegner points.

After my work with Whitehouse, I considered similar problems in higher rank. First, Furusawa and I developed a relative trace formula approach to study central values of GSp(4) L-functions in a series of papers (one with Shalika) from 2011 to 2014. This established special cases of Gan–Gross–Prasad conjectures. Second, in a joint paper with Feigon and Whitehouse (*Israel J Math*, 2018) I used relative trace formula methods to study GL($2n$) L-functions, and proved some cases of conjectures of Guo–Jacquet and Prasad.

2. Quaternionic modular forms. Under certain sign conditions, the toric periods arising in Waldspurger's formula and my 2009 *IMRN* paper are periods on modular forms on definite quaternion algebras. Around 2014–2015, I began exploring what the arithmetic of quaternion algebras tells us about periods, and thus L-values.

The first result of this pursuit was a new approach to constructing Eisenstein congruences of modular forms (which then gives congruences of L-values), which appeared in my 2017 *MRL* paper. This extended, and gave an new proof of, earlier Eisenstein congruence results of Mazur, Ribet and Yoo, to non-squarefree levels. In my 2018 *Canadian J.* paper, I used similar ideas to construct many mod 2 congruences.

To understand my construction of congruences better for general levels, I needed a concrete understanding of the structure of quaternionic modular forms, analogous to the Atkin–Lehner decomposition of classical modular forms. I carried this out in my 2020 paper in *Trans. AMS*, and as a consequence, also solved Eichler's basis problem for Hilbert modular forms.

In a recent preprint (under revision), I more directly tied my study of quaternionic modular forms to my original intent, by using the arithmetic of quaternion algebras to get exact formulas for average L-values, generalizing some work of Michel–Ramakrishnan and Feigon–Whitehouse.

3. Computational number theory. In computing various examples of quaternionic modular forms, I observed certain statistical phenomena of both classical and quaternionic modular forms. This prompted me to conduct computational investigations of modular forms and related object, leading to various conjectures. For instance, my 2020 *J. Number Theory* paper with Wiebe formulates conjectures about the frequency of zeroes of modular forms, and discusses applications to L-values. My 2021 *Proc. AMS* paper formulates conjectures about Galois orbits of modular forms, with applications to minimalist conjectures on ranks of modular forms.