PLAN(研究計画)

First we continue to study on obtained results further.

Our studies are related to various mathematics (especially geometry). Our studies are closely related to singularity theory of diffrentiable maps, classical differential topology, 3 or 4-dimensional geometry and combinatorial objects such as graphs and polyhedra. We also challenge new topics and study the following for example.

- **Symmetric spaces and Morse functions, fold maps and more general good smooth maps whose codimensions are negative.** Symmetric spaces have good symmetries. They are studied actively in differential geometry. In differential topology, their antipodal sets, which are finite sets having various essential information, have much information on the homology groups. They are in considerable cases realized as the sets of all singular points of Morse functions. Differential topological studies on symmetric spaces are also immature. We believe that higher dimensional versions of Morse functions have more information on symmetric spaces. S-commutative sets, defined by Hiroshi Tamaru, extending the class of antipodal sets, seem to be compatible with round fold maps: we believe from some explicit observations. Some round fold maps we constructed are on classical Lie groups and symmetric spaces: we believe that they represent the spaces compactly well. Motivated by this, we study the following.
  - Fold maps and more general maps on symmetric spaces –construction and general theory–.
  - The definitions and properties of meaningful point sets of symmetric spaces via singularity theory of differentiable maps.
- **Fold maps on 3-dimensional closed manifolds and meanings in 3, 4 and higher dimensional manifolds.** Saeki has proved that a 3-dimensional closed and orientable manifold admits a fold map into the plane such that the restriction to the set of all singular points is an embedding if and only if it is a so-called graph manifold. We have shown that we can take the fold maps as round fold maps into the plane in 3-4 of "Paper". In 3-5 we have an interesting integer-valued invariant for graph manifolds using the Reeb spaces. We extend the theory to 3-dimensional manifolds which may not be graph manifolds. We reveal meanings in 3, 4 and higher dimensional manifolds. This is on our challenging problem: understanding higher dimensional manifolds in geometric and constructive ways.
- **Construction of explicit smooth functions and their usage in several topics of differential geometry.** Studies on construction of smooth functions giving Reeb graphs isomorphic to given graphs have produced functions regarded as Morse-Bott functions and simple generalizations. For example, we believe that this can contribute to studies on isotropic functions and isotropic submanifolds in differential geometry via differential topology. We also study to contribute to such theory, seeing this from differential topological viewpoint.