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Research proposal

3D supersymmetric gauge theory and knot theory. Recently, in

- M. Manabe, S. Terashima and Y. Terashima, "The colored Jones polynomials as vortex partition functions," JHEP **12**, 197 (2021) [arXiv:2110.05662 [hep-th]]

we constructed and proposed an abelian gauge theory, that we called "knot-gauge theory", whose K-theoretic vortex partition functions give the colored Jones polynomials of knots in S^3. This provides a relation "colored Jones polynomial = K-theoretic vortex partition". In this work, for the construction, we utilized an exact formula of the A-twisted partition function (twisted index) of 3D $\mathcal{N}=2$ gauge theory on $S^2 \times S^1$ obtained by Benini-Zaffaroni in 2015, and the factorization of the A-twisted partition function into the K-theoretic vortex partitions. By Witten, the colored Jones polynomial is obtained as a Wilson loop expectation value in $SU(2)$ Chern-Simons gauge theory on S^3, and so our proposal is considered as a 3D-3D correspondence. At present, we do not have 6D origin of our knot-gauge theory and clear relation with the 3D-3D correspondences proposed by Terashima-Yamazaki and Dimofte-Gaiotto-Gukov in 2011, and it would be interesting to clarify them. About the knot-gauge theory, there are also some problems as follows. Firstly, the knot-gauge theory is labeled by tangle diagrams of knots, and as a result, it is non-trivially transformed under the Reidemeister moves. It is important to clarify whether one can understand this transformation in terms of "3D dualities". Secondly, in the relation "colored Jones polynomial = K-theoretic vortex partition", the right hand side is expected to be obtained as a generating function of Euler characteristics for moduli spaces of vortices. So, it is interesting to construct the vortex moduli spaces and provide a new geometric interpretation for the colored Jones polynomials. Thirdly, it would be interesting to introduce the parameter t of homological grading, by Dunfield-Gukov-Rasmussen, that categorifies the colored Jones polynomial, to the R-matrix. It is not known, for general knots, how the parameter t is introduced, and this is a mathematically challenging problem. Our construction of the knot-gauge theory is based on the computation of the colored Jones polynomial by the R-matrix, and this also gives an interpretation of the parameter t in the supersymmetric gauge theory.

Non-perturbative topological strings and refined topological strings. For perturbatively provided quantities in quantum field theories and string theories, it is important to understand non-perturbative corrections to them. The CEO topological recursion gives the perturbative topological string free energies on local CY3s, and in 2008, Eynard and Marino proposed a method to provide non-perturbative corrections. (As an application of the proposal, for obtaining the colored Jones polynomials by the CEO topological recursion summarized in "Research Accomplishments", we needed to introduce a "correction factor" by hand, but Borot and Eynard showed that it can be explained by the non-perturbative corrections.)

On the other hand, it is also known that, for the refined topological string theory on local toric CY3s, the Nekrasov-Shatashvili limit provides a non-perturbative correction to the topological string theory. (Here the refined topological string theory has two parameters ϵ_1 and ϵ_2, and the limit $\epsilon_1 = -\epsilon_2$ yields the conventional topological string theory, and the Nekrasov-Shatashvili limit is given by the limit $\epsilon_1 = 0$.) So, it would be important to clarify the relation between the non-perturbative corrections by the Nekrasov-Shatashvili limit and the ones proposed by Eynard and Marino. Also, it is known that the brane partition function in the topological string theory gives a quantization of spectral curves, and the brane partition function in the refined topological string theory is considered to give a "double quantization" of spectral curves. The relation between the double quantization and the non-perturbative corrections is an interesting problem. Along the way, it is also desirable to formulate a refined topological recursion which is so far only derived as the loop equations for the β-deformed matrix models.