(2) Study proposal Makiko Mase

(i) Aim of the study

The aim of the study is to understand algebro-geometric aspects of algebraic complex $K3$ surfaces, which we simply call a $K3$ surface . The study of geometry of $K3$ surfaces is deeply related to classical areas such as singularity theory and algebraic curve theory, and mathematical physics. It is necessary to study the period mappings and the Picard lattices for the understanding of algebro-geometric properties of $K3$ surfaces, particularly related to Picard lattices, it is unavoidable to understand the behaviour of curves on the surfaces. Related to analytic geometry and mathematical physics, it is a long-lived problem to characterize $K3$ surfaces from the view point of the moduli spaces of maps from a $K3$ surface to a Lie algebra. We consider the following problems:

Problems

1. The relation between algebro-topological properties of singularities and $K3$ surfaces.
2. The moduli space of maps from a $K3$ surface to a Lie algebra.
3. Weierstrass semi-groups of pointed curves in a $K3$ surface.

(ii) Study methods

Problem 1 This is a joint work with Professor Claus Hertling in the University of Mannheim. Our project is as follows:

- In general, it is impossible to distinguish singularities in terms of their Milnor lattice together with Seifert form on them. However, we would like to consider if a Torelli-type theorem to hold for certain special types of singularities by the Milnor lattice together with Seifert form.

- It is known that there exist 95 weight systems of $K3$ singularities, classified by Yonemura. It is also possible to relate with these singularities the $K3$ surfaces as anticanonical section of the weighted projective spaces. We are interested in comparing the Milnor lattice of $K3$ singularities with certain bilinear form and the Picard/transcendental lattices of the $K3$ surfaces. It may give a geometrical characterization of the $K3$ surface in terms of its Picard/transcendental lattices.

Problem 2 The moduli of maps are studied also in a context of singularity theory. We are interested in maps (might be with some assumption such as holomorphic) from a $K3$ surface to a Lie algebra. We expect that we have to use analytic methods to study this moduli.

Problem 3 This is a joint work with Professor Jiryo Komeda in Kanagawa Institute of Technology. We study the following questions:

- Which Weierstrass semigroups can we find for pointed curves on a $K3$ surface which is obtained by various rational elliptic surfaces ?

- Can we find other criterion for a given numerical semigroup to be Weierstrass ?

- Can we give a characterisation of numerical semigroups of type 2 ?

(iii) Aspects

The studies of singularities and of Lie algebras are to understand local properties whilst the study of $K3$ surfaces is global. But there should be deep relations between these different-looking two objects. By using the lattice theory and the algebraic geometry for subvarieties, the applicant expects that the above proposals help revealing the secret of those relations.