Research Results — Yasuo Matsushita

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1. The existence condition for a compact oriented 4-manifold to admit a neutral metric of signature $(++--)$ are settled. In relativity theory the existence condition of a Lorentz metric on a manifold is well-known. Therefore, the problem of the existence condition for the indefinite metric of neutral signature on 4-manofolds is interesting because such a metric is the lowest dimensional example of indefinite metrics not of Lorentz type. Its existence condition is known as same as the existence of a field of 2-planes. Applying Theorem of Hirzebruch and Hopf (1951) and Donaldson 's works, celebrated as the Fields Prize in 1986, the existence condition of a neutral metric on a 4-manifld is expressed in terms of the Euler characteristic and the Hirzebruch index of the manifold, which are both topological invariants. These results are cited in Donaldson's Book " The Geometry of Four-Manifolds, " published by Oxford University Press, in 1990.

2. One of the other significant results is to show a counter example constructed on a Walker 8-manifold to the Goldberg conjecture, posed in 1969, which states that an almost complex structure of a compact Einstein almost Kähler manifold must be integrable, in other words, a compact Einstein almost Kähler manifold must be Kähler.

3. Certain topological conditions on the Euler characteristics and the Hirzebruch indices on 4-dimensional compact neutral Einstein manifolds are established, which are called Thorpe-Hitchin Type Inequalities.

4. It is proved that such an existence condition of a neutral metric on a 4-manifold is equivalent to the existence condition of a pair of an almost complex structure and an opposite almost complex structure on the 4-manifold.

5. On the basis of the Enriques-Kodaira classification of compact 2-dimensional manifolds, usually called surfaces, it is shown that a surface admits an opposite almost complex structure if and only if the second Chern class of the surface is even.

6. Petean reported a new example of a neutral Einstein Kähler 4-manifold. A general method of constructing new such examples of neutral Einstein Kähler 4-manifolds are proposed in terms of arbitrary 2-dimensional harmonic functions.

7. Generalizing the notion of null vectors, we proposed isotropic tensors, with property of the zero squared norm of tensors. As one of the significant examples, a new example of isotropic Kähler structure is constructed on an Engle 4-manifold.

8. Certain detailed classifications are obtained on the basis of spinor calculus.

9. Counterexamples to the Goldberg Conjecture for an almost complex structures on eight-dimensional Walker manifolds of signature $(+4,-4)$ are constructed (2016). This is the first counterexample to the Goldberg Conjecture for indefinite metric spaces.

10. Counterexamples to the Goldberg Conjecture on six-dimensional Walker manifolds of signature $(+4,-2)$ are constructed (2015).

11. Counterexamples to the Goldberg Conjecture for an opposite almost complex structures on eight-dimensional Walker manifolds of signature $(+4,-4)$ are constructed (2016).

12. The latest results:

    It is shown that n-dimensional Walker manifolds of signature $(n-2,2)$ endowed wit a parallel field of null 2-planes admit a Kähler metric, whose three specific metric components are all harmonic functions with respect to two coordinates associated the field of null 2-planes (submitted).