Plan of Research

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1. spaces of knots in manifolds The relation between operads and embedding spaces stated in "research results" is based on embedding calculus due to Goodwillie-Weiss. This originally relates the space of embeddings and diagrams of configuration spaces and some people are studying embedding spaces other than long knots. Especially, the space of knotted circle in a manifold $\mathrm{Emb}(S^1, M)$ is being studied. In a recent preprint, I constructed a spectral sequence converging to cohomology of $\mathrm{Emb}(S^1, M)$. Comparing to Sinha's spectral sequence which abuts the same target, this spectral sequence has explicit E_2-page. I plan to calculate higher differentials of this spectral sequence. This spectral sequence was obtained by replacing the diagram of configuration spaces with a diagram of fat diagonals. Čech complex is naturally associated to a fat diagonal so elements of the spectral sequence will have an expression by a graph and cochain of the sphere tangent bundle of M. Using this expression, I will calculate the higher differentials. One can also associate a similar diagram of fat diagonals for the space of long knots in $\mathbb{R}^n$. Salvatore showed the little 2-disks operad is not formal in characteristic 2 using combinatorial model of the operad. This result is also recovered by using the diagram. I think a method similar to this proof can be applicable to calculation of higher differential for the usual long knots.

2. homotopy type of configuration spaces It is an old problem in homotopy theory whether the ordered configuration space of points in a manifold is a homotopy invariant or not. In non-simply connected case, a counter-example is known but in simply connected case, it is unsolved. Recently, Idrissi and Campos-Willwacher proved real homotopy invariance of the configuration space by constructing an algebraic model of the space. In their construction, Poincaré dg-algebra is essentially used. This is a dg-algebra which has Poincaré duality compatible with differential and product. This dg-algebra exists only in the real (or rational) coefficient and in other coefficient, any similar notion such as 'Poincaré E_∞-algebra' does not seem to be known to work. The problem is that the usual formulation of the duality on singular chain is not compatible with the product. I plan to construct an algebraic model of a manifold in any characteristic. If I manage to construct the model, I will try to prove the rational homotopy invariance of configuration spaces without transcendental methods. I will also consider the case of other coefficient.

3. geometric application of non-simply connected rational homotopy theory One of the important subject of rational homotopy theory is the restriction of homotopy type of compact Kähler manifold. In simply connected case, Deligne-Griffiths-Morgan-Sullivan shows the real homotopy type of compact Kähler manifold is formal. In non-simply connected case, Simpson gave some restriction on the fundamental group and Katzarkov-Pantev-Toën gave some restrictions on schematic homotopy type, using non-abelian Hodge theory. Tensor dg-categories naturally appear in non-abelian Hodge theory and in my formulation of rational homotopy theory gives explicit relation between models and invariants so I want to apply it to the restriction problem.