# A summary of my research

I have been studying nonlinear partial differential equations of hyperbolic and dispersive type, which have close connections with wave propagation phenomena. Of my particular interest is the wave and Schrödinger equations with critical nonlinearities, which yield a borderline between short-range and long-range situations in the small data settings. I have obtained several results on the asymptotic behavior of solutions, energy decay and non-decay, etc. In the following, the bracket [ ] indicates the paper enumerated in the separate list.

**(1) Agemi-type structural conditions for semilinear wave equations**

It is well-known that hyperbolic equations have no smoothing property. As a consequence, solutions to the Cauchy problem for nonlinear hyperbolic equations develop singularities in finite time even if the initial data are sufficiently smooth, small and having sufficiently fast decay at spatial infinity in general. So it is interesting to ask how the nonlinearity affects the singularity formation or large-time behavior of solutions to nonlinear hyperbolic equations.

The **null condition**, which is one of the most successful structural conditions, was introduced by Klainerman and Christodoulou in 1986. This guarantees the small data global existence and asymptotic free behavior at time infinity for small data solutions to three-dimensional quasilinear wave equations. Since around 2000, several weaker structural conditions than the null condition were introduced by Lindblad, Rodnianski, Alinhac, Agemi, Katayama and others. The **Agemi-type structural conditions**, which is my main research subject, is one of them. Roughly speaking, this includes not only the classical null condition but also nonlinear dissipative terms.

In the papers [1], [3] and [6], I have succeeded in extending previous results by Hoshiga (2008), Kubo (2007), Katayama-Matsumura-Sunagawa (2015), etc., from the viewpoint of energy decay/non-decay of small data solutions under the Agemi-type structural conditions. To be more precise, I have focused on the situation where the dissipative structure is partially degenerate, and obtained the following two results: (i) For single semilinear wave equation, it has been shown that the energy of the solution decays with time when the null condition is violated. Also a decay rate of the energy has been provided from the above ([3]). (ii) An example of a two-component system of the semilinear wave equations has been constructed which satisfies the Agemi-type structural conditions in which the solution has a nontrivial nonlinear effect at time infinity. In contrast to the single case, both components of the solutions to this system behave like non-trivial free solutions if the initial values of each component have a certain relation. In particular, it has turned out that the total energy of the system does not decay at time infinity ([1], [6]).

**(2) Nonlinear Schrödinger equations with weakly dissipative structure**

The Schrödinger counterpart of the Agemi-type structural condition was introduced and developed by Li-Sunagawa (2016), Sagawa-Sunagawa (2016), Sakoda-Sunagawa (2020), Katayama-Sakoda (2021) and so on.

In the papers [2], [4] and [5] (joint works with Li, Sagawa and Sunagawa), we have made some contributions to this issue. (iii) It has been shown that the $L^2$-norm of solutions for a class of derivative nonlinear Schrödinger equations under weakly dissipative structure decays at time infinity ([5]). (iv) We have constructed an example of a two-component system of the nonlinear Schrödinger equations with weakly dissipative structure whose solutions behave like non-trivial free solutions at time infinity. An important feature here is that the scattering states have a crucial restriction which never occurs if the nonlinearity were of the short-range one ([2], [4]).