

# Plan of Research

Shin'ya Okazaki

## Litherland's Alexander polynomial for handlebody-knots

A genus  $g$  handlebody-knot  $H$  is a genus  $g$  handlebody embedded in the 3-sphere. The Alexander polynomial is an invariant of a pair consisting of handlebody-knot and its meridian system. Replacing a meridian system of a handlebody-knot acts on the Alexander invariant as an action of  $GL(g, \mathbb{Z})$ . We introduced an invariant  $G_H$  for handlebody-knots which does not depend on the choice of the meridian system of  $H$  by using an invariant of the action of  $GL(g, \mathbb{Z})$  from the Alexander polynomial.

R. Litherland introduced the Alexander polynomial for  $\theta_g$ -curves. In general, the elementary ideal of the Alexander invariant is not principal for  $\theta_g$ -curves. Thus, there are infinitely many  $\theta_g$ -curves whose Alexander invariant is non-trivial and Alexander polynomial is trivial. The elementary ideal of Litherland's Alexander invariant is principal, and Litherland's Alexander polynomial is non-trivial for  $\theta_g$ -curve.

We extended Litherland's Alexander polynomial of a  $\theta_g$ -curve to that a pair of  $H$  and its meridian system with base point and understood how act replacing a meridian system for Litherland's Alexander polynomial of handlebody-knot  $4_1$ . We would like to consider that how act replacing a meridian system for Litherland's Alexander polynomial of other handlebody-knots.

## Twisted Alexander polynomial for handlebody-knots

We have some property of irreducibility of  $H$  and constituent link of  $H$  by using the Alexander polynomial as previous research. I would like to expand this result for the twisted Alexander polynomial for a handlebody-knot.

## A lower bound for the crossing number of a handlebody-knot

We have a lower bound for crossing number of constituent links of a handlebody-knot. We would like to consider a lower bound for crossing number of a handlebody-knot as an analogy.