Results of my research

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A genus 2 handlebody-knot is a genus 2 handlebody embedded in the 3-sphere, denoted by H. Two handlebody-knots are equivalent if one can be transformed into the other by an isotopy of S^3. Cutting along a meridian disk system of H, if we have knotted solid tori in S^3, then we call the spine of the knotted solid tori a constituent link of H. The constituent link depend on choice of a meridian disk. There are infinite many meridian disks for a handlebody-knot. Thus, there are infinite many constituent links for a handlebody-knot.

The degree is well known as a classical invariant of a Laurent polynomial. We introduce the following invariant as a generalization of the degree for a Laurent polynomial $f = \sum_{i=1}^{n} c_i t_1^{a_i} t_2^{b_i} \in \mathbb{Z}[t_1^{\pm 1}, t_2^{\pm 1}]$.

$$d(f) := \max \left\{ \left| \det \left(\begin{bmatrix} a_i - a_j & a_i - a_k \\ b_i - b_j & b_i - b_k \end{bmatrix} \right) \right| \;\middle|\; 1 \leq i, j, k \leq n \right\}$$

The Alexander polynomial $\Delta_{(H,M)}(t_1, t_2)$ is an invariant of a pair consisting of handlebody-knot H and its meridian system M. Replacing a meridian system of a handlebody-knot acts on the Alexander invariant as an action of $GL(2, \mathbb{Z})$. $d(\Delta_{(H,M)}(t_1, t_2))$ is an invariant for this action. Thus, the following theorem hold.

Theorem 1 [O.]
$d(\Delta_{(H,M)}(t_1, t_2))$ is an invariant for H which does not depend on M.

Y. Diao showed that the lower bound of the crossing number $c(L)$ of a link L is obtained by the degree of the Alexander polynomial as $\deg(\Delta_L(t)) \leq c(L) - b(L)$. Here, $b(L)$ is the braid index of L. The following theorem is an analogy of Diao's result.

Theorem 2 [O.]
For any constituent link L of H,

$$d(\Delta_{(H,M)}(t_1, t_2)) \leq c(L) - b(L)$$

We showed that for any $n \in \mathbb{N}$, there exists a handlebody-knot whose all of constituent links satisfy $c(L) \geq n$ by using Theorem 2.