Recently Martin Guest and C. -S. Lin introduced the $tt^*$-Toda equation and constructed global smooth solutions to the $tt^*$-Toda equation. The Tzitzéica equation appears as a special case of the $tt^*$-Toda equation, and therefore their solutions give (in special cases) special Lagrangian cones in $\mathbb{C}^3$ from my previous result. I am planning to study the properties of their solution in more details and understand a global geometry of the associated Lagrangian cones. Guest and Lin constructed solutions via an analytic method; I want to find a more geometric method based on loop groups and Iwasawa factorization. Meanwhile, Guest-Its-Lin studied the Stokes phenomena associated to the $tt^*$-Toda equation and constructed a solution by solving the Riemann-Hilbert problem. This is also very interesting.

As mentioned, I will firstly aim to construct special Lagrangian cones in $Q_0^n$ by the analogy of the previous result. It can be easily seen that special Lagrangian cones of $Q_0^n$ locally correspond to minimal Lagrangian immersions $\psi : S \to S^2 \times S^2$; therefore, obtaining the description of the initial condition for minimal Lagrangian immersions $\psi : S \to S^2 \times S^2$ in the Weierstrass representation, we can have special Lagrangian cones in $Q_0^n$. In this case, the existence of the corresponding integrable systems interests us. Additionally, it seems to be able to have the parallel discussion on the minimal Legendrian immersions into the Sasaki-Einstein manifolds associated with $S^2 \times S^2$.

The 1-forms on $S$ for minimal Lagrangian immersions $\psi : S \to \mathbb{C}\mathrm{P}^2$, hence special Lagrangian cones of $\mathbb{C}^3$, take values in certain class of the loop algebra of $\mathrm{SU}(3)$. This fact shows that some class of loop subgroups of the loop group of $\mathrm{SU}(3)$ corresponds to special Lagrangian cones in $\mathbb{C}^3$. In this project, I would like to consider properties of this class of loop subgroups.

Though loop groups are infinite-dimensional manifolds, their geometrical properties are well-known since Fourier analysis and the theory of Lie algebras are available. In particular, based loop groups are well-studied; for example, their curvatures and characteristic classes are obtained by Chern-Weil theory and the index theorem (D. Freed, *The geometry of loop groups*, 1988). Actually, various methods in differential geometry seem to be available to study of loop groups. I will try to characterize the loop subgroup for special Lagrangian submanifolds.

In 3-dimensional case, we can see that there is the equivalence between some homogeneous space coming from special Lagrangian cones, and that coming from the affine Toda equations. It can be expected that the difference is written in terms of loop groups if we can describe the correspondence of special Lagrangian cones with loop groups.

The connection of special Lagrangian cones with integrable systems depends on the fact that domains of the corresponding harmonic maps are 2-dimensional; then it is not clear that there exists a parallel relationship in higher dimensions.

On the other hand, homogeneous spaces derived from higher dimensional special Lagrangian cones can be defined. Hence, it can be discuss the correspondences to higher dimensional special Lagrangian cones and their integrability if we have a description of differences in homogeneous spaces which come from 3-dimensional special Lagrangian cones and integrable systems respectively.